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Simulation and Visualization of Safing Sensor

Khaled M. Furati, Hattan Tawfiq and Abul Hasan Siddiqi
Department of Mathematical Sciences,
King Fahd University of Petroleum and Minerals Dhahran 31261, Saudi Arabia

Abstract: Numerical simulations of Maxwell’s equations have attracted attention of several mathematicians and engineers in the recent past. Such solutions are required for proper understanding of industrial problems. In this paper we focus our attention on numerical simulation of safing sensor required by a car industry. Maxwell’s equation system models this industrial problem. We discuss numerical simulation by the fastful developed in the CISRO, Australia and released by NAG, Oxford. We take eddy current into consideration in the modeling process of saving sensor and show that finding the solution is equivalent to the numerical simulation of the Helmholtz equation. We present numerical simulation of the Helmholtz equation by the fistful. We also discuss certain results which may prove useful for further investigation of the safing sensor problem.

Key words: Safing sensor, Maxwell’s equations, Helmholtz equation, fastflo, multigrid algorithm

INTRODUCTION

Field quantities are denoted as follows:

E denote electric field intensity (V m^{-1})
D denote electric flux density (C m^{-2}
H denote magnetic field intensity (A m^{-1})
B denote magnetic flux density (Wb m^{-2})
J denote electric current density (A m^{-2})
ρ denote electric charge density (C m^{-3})

Let *C* denote any closed path, S a surface and V a region. The constitutive relationships for linear and isotropic media are:

$$J = \sigma E$$
$$B = \mu H$$
$$D = \varepsilon E,$$

where, σ is the conductivity (siemens/metre, S m^{-1}), ε is the permittivity (farad/meter, F m^{-1}) and μ is the permeability (henry/meter, H m^{-1}).

Maxwell's equations for a linear, isotropic, homogeneous medium are given as follows:

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (1.1)$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \quad (1.2)$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon} \quad (1.3)$$

$$\nabla \cdot H = 0 \quad (1.4)$$

Maxwell's equations for static fields take the form:

$$\nabla \times E = 0 \quad (1.5)$$

$$\nabla \times H = J \quad (1.6)$$

$$\nabla \cdot D = \rho \quad (1.7)$$

$$\nabla \cdot B = 0. \quad (1.8)$$

A nonlinear medium is characterized by the following nonlinear relations: $J = J(E),\ B = B(H),\ D = D(E)$. An anisotropic medium is a medium for which either J and E are parallel or B and H are not parallel or D and E are not parallel; and scalars σ, μ, ε become tensors. The properties of an inhomogeneous medium are different at different points; that is, the material constants become functions of the spatial coordinates. For more details of Maxwell's equations, we refer to[1-4].

2. Numerical simulation of safing sensor: In the airbag system of a car, there are two sensors controlling the blowing up of the airbag. The first sensor is a purely electronic sensor. This sensor may also react to electromechanical influences not caused by a car crash. To avoid the blowing up of the airbag in the non-crash cases, a second sensor is used. This is the so-called safing sensor. It is built on an electromagnetic basis. The airbag will blow up only if both sensors react. Safing sensor consists of a canonical magnet in a cylindrical form. Around this cylinder, there is a metallic ring that can move along the cylinder. Axel Klar, presently a Professor at the Technical University,

Corresponding Author: Abul Hasan Siddiqi, Department of Mathematical Sciences, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

Darmstadt, Germany, had investigated this problem a couple of years ago while working on an industrial project of the Frauenhofen Institute of Industrial and Business Mathematics, Kaiserslautern, Germany. Neunzert [Neunzert and Siddiqi[5]] has proposed an advanced model of this study through Maxwell's equations where he is mainly interested in macroscopic effects, that is in the magnetization and the forces induced by it.

It has been indicated[4] that in order to solve this problem one has to solve the following form of the Maxwell's equations:

$$\text{div } H = -\text{ div } M_K - \text{div } M_R \tag{2.1}$$

$$B = \mu H + \mu(M_K + M_R) \tag{2.2}$$

$$M_R = \mu B, \tag{2.3}$$

where, H is a magnetic field, B is magnetic induction, μ is well known constant called permeability, M is the magnetization of the cone K (magnetizati (n is a vector field M(x)) denoted by M_K and M_R the induced magnetization of the ring R. M_K is known while M_R is unknown. M_K is constant in K and it is in the direction of the axis of the magnet, that is, of the x-axis:

$$M_K(x,\ y,\ z) = (M_1,\ 0,\ 0).$$

The stationary Maxwell equation is given as:

$$\text{div } \mathbf{B} = 0 \tag{2.4}$$

Or:

$$\text{div } H = -\text{div } M$$

If we could determine H, we would get:

$$B = \mu H + \mu M.$$

B now generates the magnetization of the ring and:

$$M_R = \mu B_R$$

From (1.6) it is clear that in a region free of electric currents we have:

$$\begin{aligned} &\nabla \times H = 0 \\ &\text{div } H = \frac{1}{\mu}\text{ div } B - \mu(\text{div } M_K + \text{div } M_R) \\ &\qquad = 0 \text{ as } M_K = (M,0,0), \\ &\text{div } M_R = \text{div}\left(\frac{1}{\mu}B_R\right) = 0. \end{aligned} \tag{2.5}$$

In view of Theorem 3.38[4], there is a magnetic vector potential A such that

$$B = \nabla \times A. \tag{2.6}$$

Since $B = \mu H$, by (2.5):

$$\nabla \times B = 0. \tag{2.7}$$

By (2.6) and (2.7), we obtain:

$$\nabla \times \nabla \times A = 0. \tag{2.8}$$

Applying the formula[3]:

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla A \tag{2.9}$$

and imposing $\nabla \cdot A = 0$ (Coulomb's gauge) by (2.8), we have:

$$\Delta A = 0. \tag{2.10}$$

Since:

$$\text{div } H = \nabla \cdot H = 0, \tag{2.11}$$

employing the formula $\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \Delta H$, we obtain by (2.5) and (2.11) that:

$$\Delta H = 0. \tag{2.12}$$

By solving (2.12), we get H and so B and consequently (2.3) gives us M_R. Simulation of (2.12) by fastflo is given in Appendix-A for a metallic circular ring.

3. Numerical solution of safing sensor when the source of the field varies sinusoidally with time: An electric current induced within the body of the conductor, when the conductor either moves through a non-uniform magnitude field or is in a region where there is a change in magnetic flux, is known as the eddy current. Since iron is a conductor, there will be an eddy current in it, when the flux is changed. In this section we take into account in the airbag sensor the eddy current and carry out numerical simulation using fastflo where the metallic ring is replaced by metallic cylindrical cavity.

When the source of the field varies sinusoidally with time, the magnetic field intensity will be represented by the phaser:

$$H = H_0 \exp(j\omega t)$$

where, ω is the angular frequency and H_0 is the amplitude of the magnetic field intensity. The corresponding time-varying physical quantity is the real part of the phaser.

In the present case, the safing sensor is governed by Maxwell's equations:

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3.1)$$

$$\nabla \cdot B = 0 \quad (3.2)$$

$$\nabla \times H = J, \quad (3.3)$$

where, $J = \sigma E$, $H = \nu B$ $(\nu = 1/\mu)$, time-varying currents are distributed with density J. The medium is characterized by a permittivity ε, a reluctivity ν and a conductivity σ.

Let us also assume that J varies with time but that the time variations are small; more precisely, if time harmonic variations occur, the angular frequency ω is much lower than σ/ε.

It is clear that:

$$\nabla \times H = \sigma E. \quad (3.4)$$

Evaluating E from (3.4) and substituting it into (3.1) and using H=νB yields:

$$\nabla \times E = \nabla \times \left(\frac{1}{\sigma} \nabla \times H \right) = \nabla \times \left(\frac{1}{\sigma} \times \nu B \right)$$

Or:

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3.5)$$

By applying the identity[3]:

$$\nabla \times (\nabla \times B) = \nabla(\nabla \cdot B) - \Delta B,$$

and (1.8), we obtain from (eq: 3.5) for a homogeneous medium:

$$\Delta B = \mu\sigma \frac{\partial B}{\partial t}. \quad (3.6)$$

(3.6) is a diffusion equation and governs the magnetic field in conductive media.

Now we describe the same phenomena in terms of a vector potential A. In view of (2.4), B is solenoidal (it has zero divergence), and we can introduce a vector potential A (when it exists) by:

$$B = \nabla \times A.$$

By (3.5), we have:

$$\nabla \times \left(E + \frac{\partial A}{\partial t} \right) = 0 \quad (3.7)$$

This implies that it is possible to express the vector as the gradient of a scalar potential e by:

$$E + \frac{\partial A}{\partial t} = -\nabla \varphi \quad (3.8)$$

Therefore the current density J is:

$$J = \sigma E = \sigma \left(-\nabla \varphi - \frac{\partial A}{\partial t} \right) \quad (3.9)$$

The current expressed by equation (3.9) has two terms: a source term and an induced term. By substituting the value of H in the term of B $(H = \nu B)$ in (2.8) and taking into account $B = \nabla \times A$ and (3.3), we obtain:

$$\nabla \times H = \nabla \times \nu B = \nabla \times \nu \nabla \times A = \sigma \left(-\nabla \phi - \frac{\partial A}{\partial t} \right) \quad (3.10)$$

Since $\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \Delta A$, (3.10) gives:

$$\nu \nabla(\nabla \cdot A) - \nu \Delta A = \sigma \left(-\nabla \phi - \frac{\partial A}{\partial t} \right) \quad (3.11)$$

Or:

$$\nu \Delta A - \sigma \frac{\partial A}{\partial t} = \nu \nabla(\nabla \cdot A) + \sigma \nabla \phi \quad (3.12)$$

By introducing Coulomb's gauge $\nabla \cdot A = 0$, and denoting the source current density by $J_s = -\sigma \nabla \phi$, we finally arrive at:

$$\sigma \frac{\partial A}{\partial t} - \nu \Delta A = J_s \quad (3.13)$$

(3.13) is a parabolic differential equation for describing *A* the so-called eddy-current phenomena in conducting media subject to variable magnetization. One should add boundary and initial conditions in (3.13).

If the domain Ω is two-dimensional and rectangular coordinates are used, then as we know:

$$A = (0, 0, a), \quad J = (0, 0, j).$$

As a consequence, (3.13) reduces to:

$$\sigma \frac{\partial A}{\partial t} - \nu \Delta A = j. \quad (3.14)$$

When the sources of the field vary sinusoidally with time, so do the quantities describing the field. The magnetic field intensity will be represented by the phaser:

$$H = H_0 \exp(j\omega t),$$

where, ω is the angular frequency and H_0 is the amplitude of the magnetic field intensity. The corresponding time-varying physical quantity is the real part of this phaser. As a result, Maxwell's equation (3.1) and (3.3) become:

$$\nabla \times E = -j\omega B$$
$$\nabla \times H = J,$$

and correspondingly equations (3.6) and (3.13) reduce, respectively, to $\Delta B = j\omega\mu\sigma B$ and:

$$-\nu\Delta A + j\omega\sigma A = J \qquad (3.15)$$

(3.15) is a Helmholtz equation. Simulation of (3.15) by fastflo is given in Appendix-B.

4. A review of current results useful for safing sensor simulation: Monk and Zhang[2] have analyzed the use of edge finite elements and the multigrid method to approximate the problem of computing a static magnetic field in a cavity. Arnold, Falk and Winter[6] have constructed domain decomposition preconditioners for a positive-definite symmetric operator, which arises from the finite element discretizatgion of the boundary value problem associated with the safing sensor model.

In 1998, Hiptmair[7] adapted multigrid ideas to Maxwell's equations in general form for edge elements and in the case of discontinuous coefficients. He has studied in the cited paper solution of discrete variational problems related to the bilinear form $\langle \text{curl.}, \text{curl.}\rangle_{L_2(\Omega)} + \langle \cdot,\cdot \rangle_{L_2(\Omega)}$ defined on $H_0(\text{curl},\Omega)$. A multigrid method for the fast iterative solution of the resulting linear system has been constructed. Arnold, Falk, Winter[6] have considered the solution of the linear algebraic equations which arise from the finite element discretization of variational problems posed in the Hilbert spaces $H(\text{div})$ and $H(\text{curl})$ in three dimensions. It is shown that under appropriate conditions the multigrid *V*-cycle is an efficient solver and preconditioner for the discrete operator. Toselli[8] has developed two iterative subtracting methods for Maxwell's equations with discontinuous coefficients in two dimensions. For comprehensive properties of Hilbert spaces $H(\text{div};\Omega)$ and $H(\text{curl};\Omega)$ and their bilinear forms, we refer to[1-5]. Ammari and Nédélec[9] have given a simple new variation proof of the convergence of the electric and magnetic field solutions of the scattering problem for the Maxwell equations as the frequency goes to zero.

Edlund, Lötsted, Strand[10] has developed a hybrid method for the solution of Maxwell's equations in the frequency domain. The equations are discretized by a Galerkin method and solved by an iterative block Gauss-Seidel method. They have studied convergence of the iterations theoretically and through the numerical experiments. In 2004, Gopalkrishnan, Pasciak and Demkowicz[11] have studied a multigrid algorithm suitable for the efficient solution of indefinite linear systems arising from finite element discretization of time harmonic Maxwell equations. Efficient solution of the system of linear equations obtained by finite element discretization was a challenging problem for a long time, mainly due to the fact that linear systems are indefinite, and the differential operator curl has a large null space. In this paper a multigrid algorithm for this problem is developed. Smoothers are defined and analyzed. Convergence estimates for the multigrid algorithm are given and the results are tested by numerical experiments. Smoothing operators are based on a generalized block Jacobi or block Gauss-Seidel iteration. Lowest order Nedelec elements on cubes are used for numerical experiments.

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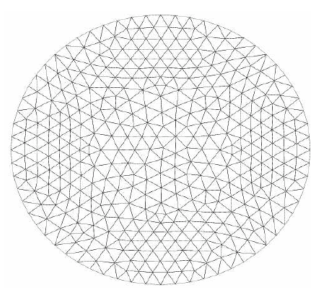
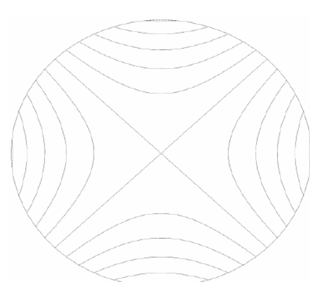
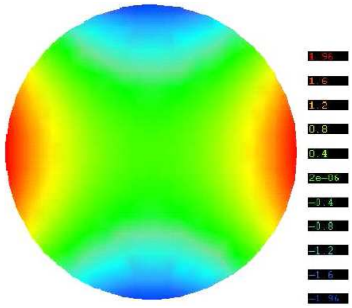
Appendix A: Simulation of Safing Sensor with Metallic Ring *fastflo* is a finite element package for the numerical solution of partial differential equations (PDEs). It is developed in collaboration between the CSIRO Mathematical and Information Sciences, Compumod, and BHP Research. Simulation of $\Delta H=0$ for metallic ring. We solve the Laplace equation on a unit circle where the value of the boundary is set to $x^2 + y^2$:



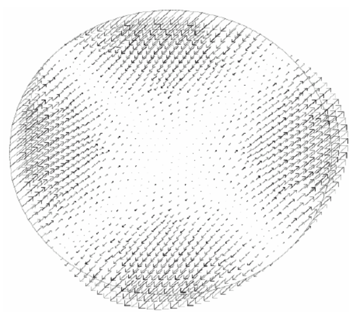
Mesh:



Solution contour



Solution shade



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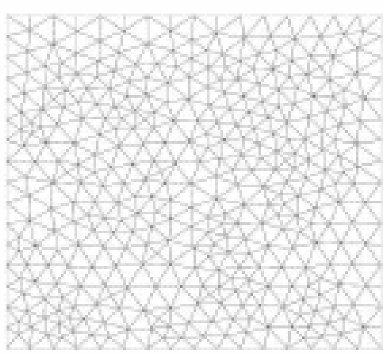
Appendix B: Simulation of safing sensor with metallic cylindrical cavity taking eddy current into consideration

We solve the Helmholtz equation on the unit square where we set J, v, j, ω, σ to 1 and for the boundary conditions we set the bottom to 1 , the top side to 3 and the two sides in 2 .



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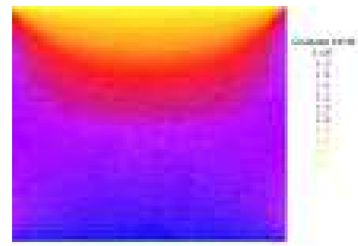
Solution contour



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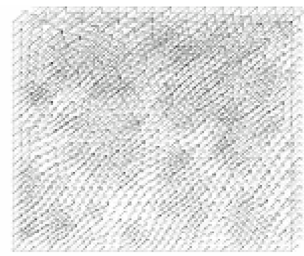

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