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Analysis of the Application of Turbulence Models in the Calculation of Supersonic Gas Jet

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Abstract: This article considers the adequacy of the usage of various two-parameter differential turbulence models for the calculation of supersonic non-calculative (overexpanded and underexpanded) gas jet flowing into the environment that have a pressure equal to atmospheric. A brief historical background on the development of methods for calculating turbulent supersonic jet, which contain developed shock wave structures, is given. For each considered turbulence model the basic relations, the values of empirical constants, general advice and experience of their usage for the calculation of various flows are provided. According to the results of tests the turbulence model that showed the best results are selected, the failures of the other models are explained. This study may be of interest to engineers using computational packages of gas-dynamics to calculate aerospace products.

Keywords: Turbulence Model, Numerical Method, Supersonic Gas Jet

Introduction

It is important to determine the intensity of Difference modeling of jet streams-the main method of studying the supersonic gas jets. The most comprehensive overview of the research history in this field is given in the monumental works of (Fletcher, 1991; Anderson *et al*., 1990). The development of applied methods for calculating supersonic jet of aircraft is represented in the collection (Nilsen, 1988), written by leading experts of the U.S. aerospace industry. Note also the monograph, written by Avduevsky *et al*. (1989), which provides an overview of classical approximate and semi-empirical methods of ideal gas jets calculation.

Methods with the capture of Gas-Dynamic Discontinuities (GDD) in the 80 s were the main way to increase the accuracy of the calculation. Significant step forward in their development was when Adrianov A.L. suggested to carry out the capture of shock-wave structure elements with the help of exact solving of the zero and first order problems on the interaction of the strong and weak of gas-dynamic discontinuities.

Zonal calculation methods involve dividing the entire flow field into separate elements (zones) and the application of the most suitable algorithm to each of them with subsequent joining of the solutions.

Parabolized Navier-Stokes Equations (PNSE) appear to be an attractive model in cases where there is a certain preferred direction, along which vary the gas-dynamic variables, for example, it may be an symmetry axis of jet or aircraft. Development of numerical methods for calculating real gas jets using PNSE can be traced to the works of (Dash and Thorpe, 1981; Dash *et al*., 1985a; 1985b; 1985c; 1985d), aimed at the creation of a working methodology for calculating the plume of a solid-fuel tactical missile. One of the few studies, in which the tail part of the missile with the jet is correctly calculated is the article of (Sahu, 1987).

The main difficulties in the application of numerical methods, even the most committed, based on the zonal approach, the allocation of GDD and usage of PNSE are associated with tracking of inceptive hanging shocks and calculating subsonic areas behind the Mach disk. Around the second half of the 90 s, a new trend associated with the improvement of turbulence models has been formed. Only at the end of the first decade of the XXI century a tangible successes appeared in this area.

Differential Model of Turbulence

The most detailed overview of modern turbulence models in the annex to the actual calculation of the jet engine is given in the monograph of (Volkov and Emelyanov, 2010). As in several other review articles (Snegiryov, 2009) the subsonic and transonic flows are mainly considered.

The basis for all models with the two equations is the hypothesis of Boussinesq about turbulent (vortical) viscosity, which states that the Reynolds stress tensor is proportional to the arithmetic mean value τijSij of the strain rate tensor and can be written in the form of:

$$\tau_{ij} = 2\mu_i S_{ij} - \frac{2}{3}\rho k \delta_{ij},$$

where, μ-a scalar value, called the turbulent (vortical) viscosity, which is usually calculated μt from the two transport variables.

The last term is included for simulation of incompressible flows to satisfy the definition of the kinetic energy of turbulence:

$$k = \frac{\overline{u_i' u_j'}}{2}.$$

In more generalized form this equation may be rewritten as follows:

$$-\rho\overline{u_i' u_j'} = \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\rho k \delta_{ij},$$

Boussinesq hypothesis is both strong and weak side of the model with two equations. This hypothesis is rough assumption that turbulence affects the averaged flow in the same way as the molecular viscosity affects the laminar flow. The hypothesis also allows us to introduce scalar variables intuitively characterizing the turbulence, such as turbulent viscosity and turbulence energy dissipation and associate them with even more intuitive variables like the turbulence intensity and the characteristic scale of turbulence.

Weak side of the Boussinesq hypothesis is the fact that it is generally not entirely justified. Nothing indicates that the Reynolds stress tensor must be proportional to the strain rate tensor. This is true for simple flows, such as flat boundary layers and near wake of the flow around the bodies, but in complex flows, such as flows with strong curvature of streamlines and rapidly accelerating or retarding flows, the Boussinesq assumption is simply unacceptable. This creates problems with the calculation of strongly rotating flows and flows where the effects of streamlines' curvature are important. Turbulence model with two differential equations also often have problems with the calculation of strongly decelerating streams, such as stagnant flows.

K-ε Turbulence Model

K-ε model is the most common and frequently used, despite its shortcomings in the calculations under condition of large pressure gradients (Wilcox, 1998). As a model with two equations, it includes two additional transport equations, allowing to explain effects such as convection and diffusion of turbulent energy.

Initial push to the creation of k-ε model was the aim to improve the algebraic model of the mixing length (Wilcox, 1988), as well as the search for alternative ways to algebraically describe the characteristic scale of turbulence in the flows of middle and a high degree of complexity. The experiments showed the reduction in accuracy for flows with large pressure gradients, thus it was concluded that this model can be most effectively used solving the problems of internal and wall-adjusted flows with medium pressure (Bardina *et al*., 1997). Like in models with a single equation a linear scale is needed to describe the effect of the walls. There are two different approaches to the definition of the linear scale in the forming of such functions.

Within the scope of first approach, the distance to the wall is used as a linear scale. In the second approach, the linear scale is constructed without using the distance to the wall and is based on the turbulent characteristics (e.g., the Reynolds turbulent number Re = k^2/Vε).

There are numerous k-ε models, the most widespread of which is the model of Launder-Sharma.

Below are the basic relations for the standard k-ε model:

Transport equation of the standard k-ε model for the turbulent kinetic energy k:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k + P_b - \rho\varepsilon - Y_M + S_k$$

For the dissipation rate of turbulent energy ε:

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}(P_k + C_{3\varepsilon}P_b) - C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} + S_\varepsilon$$

Turbulent viscosity:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$$

Generation of turbulent kinetic energy:

$$P = -\rho\overline{u_i' u_j'}\frac{\partial u_j}{\partial x_i} P_k = \mu_t S^2$$

where, S-absolute value of strain rate tensor size:

$$S \equiv \sqrt{2S_{ij}S_{ij}}$$

Influence of buoyancy:

$$P = \beta g_i \frac{\mu_t}{Pr_t}\frac{\partial T}{\partial x}$$

where, Pr_t-Prandtl turbulent number for energy and g_i-a component of the gravity vector in i-m direction. For the standard and realizable model the standard value Pr_t = 85.

Thermal expansion coefficient β:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Constants of the model: $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$, $C_\mu = 0.09$, $\sigma_k = 10$, $\sigma_\varepsilon = 1.3$.

K-E Realizable Model

The model k-ε realizable proposed in the work of (Yakhot *et al*., 1992). This model introduces an improved method of calculating turbulent viscosity and the equation for the dissipation rate is derived directly from the exact transport equation of vorticity fluctuating component's rms value.

The term "Realizable" means that the model satisfies the mathematical constraints on the normal stresses, consistent with the physics of turbulent flows (negative values of the vortical viscosity are excluded in the calculation of the high-gradient flows). This is achieved by introducing a functional dependence of C_μ coefficient. Compared to the standard version the model k-ε realizable can most accurately predicts the dissipation rate of flat and round jets and also provides better prediction of boundary layers characteristics, subjected to strong pressure gradients, separated and recirculating flows, as well as flows in which the developed secondary flow exist.

Let's consider the basic relations for the Realizable model.

Transport equation:

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_j}(\rho\varepsilon u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + \rho C_1 S\varepsilon - \rho C_2 \frac{\varepsilon^2}{k + \sqrt{v\varepsilon}} + C_{1\varepsilon}P_b + S_\varepsilon$$

Where:

$$C_1 = max\left[0.43\frac{\eta}{\eta+5}\right], S = \sqrt{2S_{ij}S_{ij}}.$$

In these equations, P_k represents the formation of turbulent kinetic energy due to average velocity gradients, calculated in the same way as in the standard k-ε model. P_b-is the formation of turbulent kinetic energy due to buoyancy, calculated in the same way as in the standard k-ε model as well.

Turbulent viscosity:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$$

Where:

$$C_\mu = \frac{1}{A_0 + A_S\frac{kU^*}{\varepsilon}}; U^* \equiv \sqrt{S_{ij}S_{ij} + \Omega_{ij}\Omega_{ij}}; \Omega_{ij} = \Omega_{ij} - 2\varepsilon_{ijk}\omega_k; \Omega_{ij} = \overline{\Omega_{ij}} - \varepsilon_{ijk}\omega_k.$$

Where:

$\overline{\Omega_{ij}}$ -rotation velocity average tensor, considered in the frame, rotating with angular velocity ω_k.

Model constants A_0 and A_s: $A_0 = 4.04$, $A_S = \sqrt{6}\cos\phi.$:

$$\phi = \frac{1}{3}\cos^{-1}\left(\sqrt{6}W\right), W = \frac{S_{ij}S_{jk}S_{ki}}{\tilde{S}^3}, \tilde{S} = \sqrt{S_{ij}S_{ij}}, S_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right).$$

Model constants: $C_{1\varepsilon} = 1.44$, $C_2 = 1.9$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.2$.

Rng K-E Model

RNG approach is a special mathematical technique that can be used to formulate a model of turbulence, similar to the k-ε model. By modifying the ε- equation an attempt to take into account the existence of different turbulent motion scales is made.

RNG k-ε model was developed using the methods of Re-Normalisation Group (RNG) by (Yakhot *et al*., 1992) to renormalize the Navier-Stokes equations and calculate the effects of small-scale motion. In the standard k-ε model the turbulent viscosity is derived from a single characteristic linear scale of turbulence, so the calculated turbulent diffusion-that's just what happens in this scale, whereas, in reality, all scales of motion will contribute to turbulent diffusion.

There are several ways to describe the transport equations k and ε, for example, in the absence of buoyancy:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \rho\varepsilon$$

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}P_k - C^*_{2\varepsilon}\rho\frac{\varepsilon^2}{k}$$

Where:

$$C^*_{2\varepsilon} = C_{2\varepsilon} + \frac{C_\mu \eta^3 (1 - \eta/\eta_0)}{1 + \beta\eta^3}$$

$$\eta = Sk/\varepsilon$$

$$S = \left(2S_{ij}S_{ij}\right)^{1/2}$$

Turbulent viscosity can be calculated in the same way as in the standard k-ε model.

It is interesting to note that the values of all constants (except β) are obtained explicitly in the RNG procedure. Theoretical values of coefficients in comparison with the empirical values in the standard k-ε model are provided below.

Table 1. RNG model coefficients

Coefficient	Theoretical value in RNG	Empirical value in standard model
C_μ	0.0845	0.09
σ_k	0.7194	1.00
σ_ε	0.7194	1.30
$C_{\varepsilon 1}$	1.4200	1.44
$C_{\varepsilon 2}$	1.6800	1.92

$\eta_0 = 4.38$; $\beta = 0.012$ (determined during the experiment)

Although the technique of obtaining RNG equations seemed revolutionary at the time of creation, its use was rather limited. Some researchers stated that the method gives better accuracy in rotating flows. However, in this matter, there was conflicting results. Method showed higher accuracy in modeling rotating cavities, but did not show any advantage over the standard method for calculating the vortex evolution. RNG model is best suited for modeling convection indoors.

K-ω Turbulence Model

K-ω model is similar to k-ε, but in it the equation for turbulent energy dissipation rate is used instead of the dissipation equation.

The first transport variable is the turbulence kinetic energy k, the second-the relative dissipation ω. It determines the typical linear scale of turbulence, while the first variable k-turbulence energy.

The combination of equations for k and ω was first proposed by Kolmogorov 1942, but his equations cannot be considered a turbulence model in the modern sense of the word. Active promotion of the k-ω model (as a tool for the calculation of turbulent flows) was engaged by Wilcox, he created a number of models, in particular, the co-called high Reynolds Wilcox model (Wilcox, 1988), recognized the.

K-ω model is often used for the calculation of flows in turbo-machinery. For example, General Electric Company uses it to create engines that are actively used in the Airbus.

This model is good at describing the near-wall flow, without having problems with the pressure gradient. However, when using it problems with the calculations of jet flows occur. The primary of them is the extreme sensitivity to the boundary conditions in the external flow. The modified model containing additional conditions was designed to get rid of these shortcomings.

Kinetic turbulence energy:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = P - \beta^* \rho \omega k + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \frac{\rho k}{\omega}\right)\frac{\partial k}{\partial x_j}\right]$$

Relative dissipation rate:

$$\frac{\partial(\rho\omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \frac{\gamma\omega}{k}P - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_\omega \frac{\rho k}{\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \frac{\rho\sigma_d}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

Where:

$$P = \tau_{ij}\frac{\partial u_i}{\partial x_j}$$

$$\tau_{ij} = \mu_t\left(2S_{ij} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) - \frac{2}{3}\rho k \delta_{ij}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

The turbulent eddy viscosity is computed from following:

$$\mu_t = \frac{\rho k}{\hat{\omega}}$$

Where:

$$\hat{\omega} = \max\left[\omega, C_{\lim}\sqrt{\frac{2\bar{S}ij\bar{S}ij}{\beta^*}}\right]$$

$$\bar{S}_{ij} = S_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

Since the k-ω turbulence model allows to simulate the turbulence of different scales, it has been applied in areas such as calculations of flows, where the boundary layer is essential. For example, of the turbo-machine and external flow of aircraft.

Standart SST K-ω Model and the Transition SST Model

Turbulent k-ω model Shear Stress Transport (SST) was introduced by (Menter, 1993) and immediately gained a lot of popularity. This model is essentially a combination of two (k-ε away from the walls and k-ω near the walls). Using k-ω model in the inner parts of the boundary layer allows the use of SST model directly up to the wall through the viscous sublayer.

In a free turbulent flow Menter model behaves as k-ε, thereby avoiding the usual k-ω model's problems, which is its sensitivity to the initial conditions of turbulent flow. Researchers, who use the standard SST model, typically find that it shows good results in mixing layers at moderate pressure gradients.

In areas with high normal stresses such as stagnant flow and high acceleration areas, the standard SST k-ω model generates too much turbulence levels. Nevertheless, the tendency is much lesser than normal k-ε model.

Transition SST model is based on two transport equations, one of which is for the intermittency and other-for criteria for the pressure in terms of the Reynolds number, calculated for a moment of the momentum loss thickness.

Below are the basic relations for the standard SST model, described in more detail in (Menter, 1993; 1994).

Kinematic turbulent viscosity:

$$v_T = \frac{a_1 k}{max(a_1\omega, SF_2)}$$

Kinetic energy of turbulence:

$$\frac{\partial \omega}{\partial t} + U_j\frac{\partial k}{\partial x_j} = P_k - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(v + \sigma_k v_T)\frac{\partial k}{\partial x_j}\right]$$

Relative dissipation rate:

$$\frac{\partial \omega}{\partial t} + U_j\frac{\partial \omega}{\partial x_j} = \alpha S^2 - \beta\omega^2$$
$$+\frac{\partial}{\partial x_j}\left[(v + \sigma_\omega v_T)\frac{\partial \omega}{\partial x_j}\right] + 2(1-F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$

Additional relations and constants:

$$F_2 = \tanh\left[\left[\max\left(\frac{2\sqrt{k}}{\beta^*\omega y}, \frac{500v}{y^2\omega}\right)\right]^2\right],\ P_k = \min\left(\tau_{ij}\frac{\partial U_i}{\partial x_j}, 10\beta^* k\omega\right)$$

$$F_1 = \tanh\left\{\left\{\min\left[\max\left(\frac{\sqrt{k}}{\beta^*\omega y}, \frac{500v}{y^2\omega}\right), \frac{4\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right]\right\}^4\right\},$$

$$CD_{kw} = \max\left(2\rho\sigma_{w2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, 10^{-10}\right)$$

$$\phi = \phi_1 F_1 + \phi_2(1-F_1)$$

$$\alpha_1 = \frac{5}{9},\ \alpha_2 = 0.44,\ \beta_1 = \frac{3}{40},\ \beta_2 = 0.0828,\ \beta^* = \frac{9}{100}$$
$$\sigma_{k1} = 0.85,\ \sigma_{k2} = 1,\ \sigma_{\omega 1} = 0.5,\ \sigma_{\omega 2} = 0.856$$

Transport equation for the intermittency in Transition SST model has the form:

$$\frac{\partial(py)}{\partial t} + \frac{\partial(\rho U_j y)}{\partial x_j} = P_{y1} - E_{y1} + P_{y2} - E_{y2} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_y}\right)\frac{\partial y}{\partial x_j}\right]$$

Transport equation of criteria for the pressure in terms of the Reynolds number, calculated for a moment of the momentum loss thickness:

$$\frac{\partial(\rho Re_{\theta t})}{\partial t} + \frac{\partial(\rho U_j Re_{\theta t})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu + \mu_t)\frac{\partial Re_{\theta t}}{\partial x_j}\right]$$

Standard SST model gives as good results as for the separation of the flow and for high pressure gradients. Moreover, this model has been proven reliable and does not demand much computing power. Transition SST model allows to more accurately describe the turbulence due to the introduction of additional transport equations. SST model-is the new industry standard and gives very good results even for the calculation of phenomena such as flow separation during the blowing of air in the boundary layer or in the process of turbulent heat transfer.

Results of Turbulence Models Testing

In the simulation of supersonic jets flowing from the exhaust devices of air-jet engine, the accuracy of the model depends on the degree of turbulence non-isobarity (the ratio of the static pressure at the nozzle to the static pressure in the environment) of the jet.

Authors of this study performed a systematic testing of supersonic jet flows. For the testing purposed the standard k-ε model in the formulation of the Launder-Sharma, k-ε Realizable, RNG k-ε, standard SST k-ω model and the Transition SST k-ω model, widely represented in modern commercial computing packages were selected.

Although these models are widely used in solving of a variety of engineering problems, they are still the subject of active research. Basic relations and empirical constants values of the turbulence models, listed above can be found in works of (Sahu, 1987; Volkov and Emelyanov, 2010). The best results of low underexpanded jets calculation are shown by k-ε realizable model (Fig. 1).

Shock-Wave Structure (SWS) of jet with low non-isobarity is determined mainly by the characteristics of the mixing layer at its boundary, which is again, described best by the k-ε realizable model. The effect of the areas of shocks reflection from the symmetry axis, in which the shortcomings of k-ε realizable model manifests, is much lesser. On the contrary, the standard SST k-ω model overestimates the level of turbulence in the areas of shocks interaction with mixing at the boundary layer of the jet, which leads to a more rapid erosion of SWS compared with experiment.

Position changes in the case of highly non-isobaric jets (underexpanded or overexpanded). Shocks reflect from the jet axis, generating triple points and Mach discs, flow behind which is subsonic and strongly vorticle (Bulat *et al.*, 2012). Flow in the vicinity of the triple point has large pressure gradients and high levels of turbulence of vorticity. Best qualities, when calculating such flows, are demonstrates by Transition SST model (Fig. 2a).

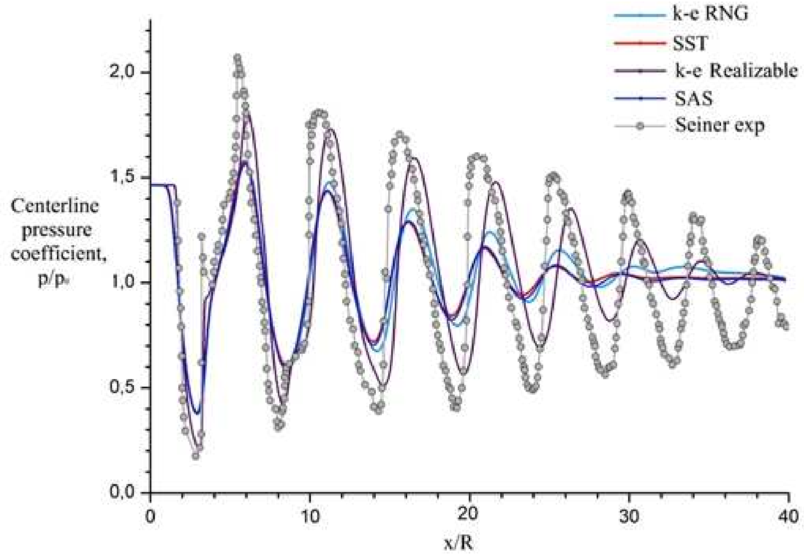

Fig. 1. Comparison of calculation results with Seiner experiment (Dash *et al*., 1985a) using different models of change in static pressure turbulence along the symmetry axis of low underexpanded jet, flowing from the profiled nozzle with a Mach number at the cut Ma = 2, amount of non-isobarity = 1.445

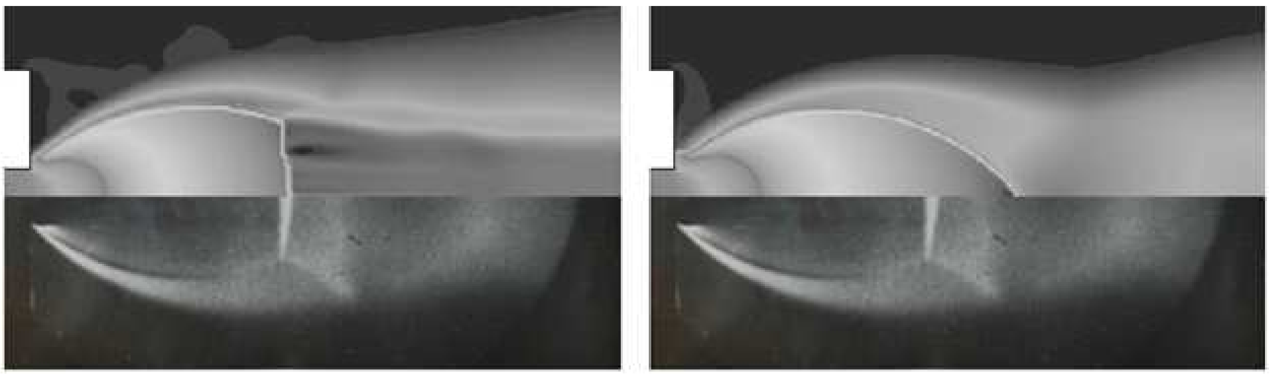

Fig. 2. Comparison of the calculation and experimental results for underexpanded jet with an nonisobarityof n = 24, flowing out of the nozzle with the Mach number on the cut $M_a = 1$.

A large diameter sonic nozzle was selected for testing to minimize the impact of nozzle shocks and boundary layer on the nozzle walls.

The usage of k-ε realizable turbulence model, on the contrary, leads to unsatisfactory results (Fig. 2b). The inherent model procedure of limiting the turbulent viscosity by introducing semi-empirical functional dependence for turbulent viscosity coefficient Cμ, leads to disruption of differential conditions of dynamic compatibility on the shock waves at the triple point. As a result, the formation of a Mach disc is prolonged and its dimensions are much smaller than the experimentally observed. Other models give intermediate results.

Conclusion

In this study a research of two-parameter differential turbulence models for the calculation of supersonic gas jets was performed. The paper provides the basic relations, the values of empirical constants, general advices and experience of their usage for the calculation of various flows for each considered turbulence model. According to the results of tests, k-ε realizable and transition SST turbulence model are selected as those, which showed the best results. The failures of the other models are explained.

Findings

Testing has shown that the best results in the calculation of supersonic flows, typical for prospective AJE are given by k-ε realizable and transition SST turbulence model. The first model, at small difference in the pressure at the nozzle exit and the in the environment, which is typical for the normal operation regimes of AJE provides reliable data on the pressure distribution along the jet axis, bottom pressure, the pressure distribution on the walls of the nozzle and ejector. It is possible to achieve a proper accuracy on a fairly coarse grid without using any special techniques and tricks. Transition SST turbulence model is more demanding to the difference grid, boundary and initial conditions and requires substantially more computation time. Instead, it allows to get reliably accurate picture of SWS, determining the pressure distribution along the axis of the jet with acceptable accuracy.

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Author's Contributions

All authors equally contributed in this work.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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