Journal of Mathematics and Statistics 1 (3): 225-233, 2005
ISSN 1549-3644

Nonlinear Growth Models for Modeling Oil Palm Yield Growth

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Abstract: This study provided the basic needs of parameters estimation for nonlinear growth model such as partial derivatives of each model, determination of initial values for each parameter and statistical tests of industrial usage. Twelve nonlinear growth models and its partial derivatives for oil palm yield growth are presented in this study. The parameters are estimated using the Marquardt iterative method of nonlinear regression relating oil palm yield growth data. The best model was selected based on the model performance and it can be used to estimate the oil palm yield at any age of oil palm. This study found that the Gompertz, logistic, log-logistic, Morgan-Mercer-Flodin and Chapman-Richard growth models have the ability for quantifying a growth phenomenon that exhibit a sigmoid pattern over time. Based on the statistical testing and goodness of fit, the best model is the Logistic model and followed by the Gompertz model, Morgan-Mercer-Flodin, Chapman-Richard (with initial stage) and Log-logistic growth models.

Key words: Nonlinear growth model, partial derivative, oil palm yield

INTRODUCTION

Growth model methodology has been widely used in the modeling of plant growth. Since growth of living things are normally nonlinear, it is reasonable to explore the use of nonlinear growth model to oil palm yield. In the oil palm industry, there are only a few theoretical model formulated specifically for oil palm industry applications. Modeling of the growth in other disciplines and application here a considerable potential for modeling of the Fresh Fruit Bunches (FFB) growth and oil palm yield. This is partly attributed to the fact that the statistical methodology used for fitting nonlinear models to oil palm growth data is closely related to the mathematics of the models and not explored yet. From our exploratory study on modeling practices, little work has been reported on modeling the oil palm yield growth. A nonlinear growth model was developed and proposed the used of partial derivatives of twelve nonlinear growth models. Growth studies in many branches of science have demonstrated that more complex nonlinear functions are justified and required if the range of the independent variable encompasses juvenile, adolescent, mature and senescent stages of growth[1]. Some of the application of the nonlinear model in agronomy was conducted by[2] is cocoa industry. While research in modeling tobacco growth data was done by[3-4].

The problem in modeling oil palm yield growth is that it does not follow a linear model. It normally follows a nonlinear growth curve. With the increase in the number of independent variables, modeling a nonlinear curve becomes more complex. This causes the model to be more inaccurate. The function of a growth curve has a sigmoid form ideally its origin is at (0,0), a point of inflection occurring early in the adolescent stage and either approaching a maximum value, an asymptote or peaking and falling in the senescent stage. Normally, oil palm will be harvested after four years of planting. The oil palm yield will increase vigorously until the tenth year of planting. The yield will then remains at a stable stage until the twenty-fifth year. The oil palm yield growth data are given in Table 1. The data used in this study are secondary data from research done by[5-6]. The research was conducted at Serting Hilir in Negeri Sembilan which normally annual rainfall in this location is had 1600mm to 1800 mm with two distinct droughts in January-March and June-August, however, the weather relatively wet. The data used here is the average of fresh fruit bunches and measured in tonnes/hectare/year from year 1979 to 1997.

Nonlinear models are more difficult to specify and estimate than linear models and the solutions are determined iteratively[7-8]. The iterative method used in the nonlinear regression model include the modified Gauss-Newton method (Taylor series), gradient or steepest-descent method, multivariate secant or false position and is the Marquardt method[7]. If a model, after reparameterization, does not behave in a near-linear fashion, the parameter estimates will not have desirable properties such as unbiasedness, normality and minimum variance and hence, complex estimation techniques (e.g[9]) may be necessary[8]. In such cases,

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Table 1: The oil palm yield data

Year	4	5	6	7	8	9	10	11	12	13
Weight	11.78	18.43	25.21	30.78	33.03	35.66	36.96	37.97	38.04	39.20
Year	14	15	16	17	18	19	20	21	22	
Weight	36.50	37.21	39.97	38.45	33.65	34.71	37.75	32.81	37.99	

the use of partial derivatives rather than computational approximations usually results in more efficient and more precise parameter estimation. Therefore, the purpose of this study was to derive the partial derivatives of twelve nonlinear growth models and demonstrate the method of parameter estimation using experimental oil palm yield growth data. The NLIN (nonlinear regression) procedure in[10] was used to fit the models to the oil palm yield growth data and estimate the parameters.

Since the nonlinear growth model study has not been explored yet in oil palm industry, this study was conducted purposely to model the oil palm yield growth using nonlinear growth model. This study provided the basic needs of parameters estimation for nonlinear growth model such as partial derivatives of each model, determination of initial values for each parameter and statistical tests of industrial usage. Twelve nonlinear growth models were considered and the equations are presented in Table 1. The best model was selected based on the model performance and it can be used to estimate the oil palm yield at any age of oil palm. Then, with this useful information of the age and the total planted area of oil palm, the model can also be used to estimate the total production of Malaysian oil palm yield.

MATERIALS AND METHOD

For a nonlinear regression model

$$\phi_i = f(t_i, \beta) + \varepsilon_I \qquad (1)$$

$i = 1, 2, \ldots, n$, where ϕ is the response variable, t is the independent variable, β is the vector of parameters β_j to be estimated $(\beta_1, \beta_2, \ldots, \beta_k)$, ε_i is a random error term, k is the number of unknown parameters and n is the number of the observation. The estimators of β_j's are found by minimizing the sum of squares error (SS_{err}) function as below

$$SS_{err} = \sum_{i=1}^{19}\left[\phi_i - f(t_i, \boldsymbol{\beta})\right]^2 \qquad (2)$$

under the assumption that the ε_i are normal and independent with mean zero and common variance σ^2. Since ϕ_i and t_i are fixed observations, the sum of squares residual is a function of β. Least squares estimates of β are values which when substituted into Eq. 2 will make the SS_{err} a minimum are found by

Table 2: Nonlinear mathematical models considered in this study

Model	Integral equation form	Source
Logistic	$\phi(t) = \alpha/(1+\beta\exp(-\kappa t)) + \varepsilon$	[7]
Gompertz	$\phi(t) = \alpha\exp(-\beta\exp(-\kappa t)) + \varepsilon$	[7]
Von Bertalanffy	$\phi(t) = [\alpha^{1-\delta} - \beta e^{-kt}]^{\frac{1}{1-\delta}} + \varepsilon$	[8, 11]
Negative Exponential	$\phi(t) = \alpha(1-\exp(-\kappa t)) + \varepsilon$	[1]
Monomolecular	$\phi(t) = \alpha(1-\beta\exp(-\kappa t)) + \varepsilon$	[7]
Log-logistic	$\phi(t) = \alpha/(1+\beta\exp(-\kappa\ln(t))) + \varepsilon$	[12]
Richard's	$\phi(t) = \alpha/\left[1+\beta\left(\exp(-\kappa t)\right)^{\frac{1}{\delta}}\right] + \varepsilon$	[8]
Weibull	$\phi(t) = \alpha - \beta\exp(-\kappa t^{\delta}) + \varepsilon$	[8, 13]
Schnute	$\phi(t) = (\alpha + \beta\exp(\kappa t))^{\delta} + \varepsilon$	[14-15]
Morgan -Mercer-Flodin	$\varphi(t)=(\beta\gamma+\alpha t^{d})/\gamma+t^{\delta} + \varepsilon$	[13, 16]
Chapman-Richards	$\phi(t) = \alpha(1-\beta\exp(-\kappa t))^{\frac{1}{1-\delta}} + \varepsilon$	[7]
Stannard	$\varphi(t)=\alpha\left[1+\exp\left(-(\beta+\kappa t/\delta_3)\right)\right]^{\delta} + \varepsilon$	[12]

differentiating Eq. 2 with respect to each parameter and setting the result to zero. This provides the k normal equations that must be solved for $\hat{\boldsymbol{\beta}}$. These normal equations take the form

$$\sum_{i=1}^{19}\{\phi_i - f(t_i, \boldsymbol{\beta})\}\left[\frac{\partial f(t_i, \boldsymbol{\beta})}{\partial \beta_j}\right] = 0 \qquad (3)$$

for $j = 1, 2, .., k$. When the model is nonlinear in the parameters so are the normal equations. Consequently, for the nonlinear models considered in Table 2, it is impossible to obtain a closed form solution to the least squares estimate of the parameters by solving the k normal equations described in Eq. 3. Hence an iterative method must be employed to minimize the SS_{err}[7-8].

The Marquardt iterative method is an estimator method, which represent a compromise between the Gauss-Newton method and the steepest descent method. It is a method that combined the best features of both while avoiding their most serious limitations. Due this characteristic we decided to use the method. The Marquardt iterative method requires specification of the names and initial values of the parameters to be estimated, the model using a single dependent variable and the partial derivatives of the model with respect to each parameter [10].

The usual statistical tests, which are appropriate in the general linear model case, are, in general, not appropriate when the model is nonlinear and one cannot

Table 3: Partial derivatives of the Logistic and Gompertz, von Bertalanffy, Negative exponential growth models

	Model and partial derivatives
	Logistic: $\phi(t) = \alpha_0/(1+\alpha_1 \exp(-\alpha_2 t)) + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= 1/(1 + \alpha_1 \exp(-\alpha_2 t))$
$\partial\phi/\partial\alpha_2$	$= (-\alpha_0 \exp(-\alpha_2 t))/(1 + \alpha_1 \exp(-\alpha_2 t))^2$
$\partial\phi/\partial\alpha_1$	$= (\alpha_0\alpha_1 t)/(1 + \alpha_1 \exp(-\alpha_2 t))^2)(\exp(-\alpha_2 t))$
	Gompertz: $\phi(t) = \alpha_0 \exp(-\alpha_1 \exp(-\alpha_2 t)) + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= \exp(\alpha_1 \exp(-\alpha_2 t))$
$\partial\phi/\partial\alpha_2$	$= -\alpha_0 \exp(-\alpha_1 \exp(-\alpha_2 t))(\exp(-\alpha_2 t))$
$\partial\phi/\partial\alpha_1$	$= \alpha_0\alpha 1t \exp(-\alpha_1 \exp(-\alpha_2 t))(\exp(-\alpha_2 t))$
	Von Bertalanffy: $\phi(t) = \left[\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\ \right]^{\frac{1}{1-\alpha_3}} + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= (\alpha_0^{-\alpha_3})\left[\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\right]^{\frac{1}{(1-\alpha_3)}-1}$
$\partial\phi/\partial\alpha_3$	$= \left(-\exp(-\alpha_2 t)/(1-\alpha_3)\right)\left(\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\right)^{\frac{1}{(1-\alpha_3)}-1}$
$\partial\phi/\partial\alpha_1$	$= \left(\alpha_1 t/(1-\alpha_3)\right)\left(\exp(-\alpha_2 t)\right)\left[\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\right]^{\frac{1}{(1-\alpha_3)}-1}$
$\partial\phi/\partial\alpha_2$	$= \left\{\exp\left((1/(1-\alpha_3))\ln\left(\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\right)^{\frac{1}{(1-\alpha_3)}-1}\right)\Big/(1-\alpha_3)\right\} *$
	$\left\{\begin{array}{l}\left(\ln\left(\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\right)/(1-\alpha_3)\right) - \\ \left(\ln(\alpha_0)\left(\alpha_0^{(1-\alpha_3)}\right)/\left(\alpha_0^{(1-\alpha_3)} - \alpha_1 \exp(-\alpha_2 t)\right)\right)\end{array}\right\}$
	Negative exponential: $\phi(t) = \alpha_0(1-\exp(-\alpha_2 t)) + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= (1-\exp(-\alpha_2 t))$
$\partial\phi/\partial\alpha_1$	- does not exist
$\partial\phi/\partial\alpha_2$	$= (\alpha_0 t \exp(-\alpha_2 t))$

Table 4: Partial derivatives of the Monomolecular, log-logistic, Richard's, Weibull, Schnute Morgan-Mercer-Flodin gowth models

	Monomolecular: $\phi(t) = \alpha_0(1-\alpha_1 \exp(-\alpha_2 t)) + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= (1- \alpha_1 \exp(-\alpha_2 t))$
$\partial\phi/\partial\alpha_1$	$= (-\alpha_0 \exp(-\alpha_2 t))$
$\partial\phi/\partial\alpha_2$	$= (\alpha_0\alpha_1 t \exp(-\alpha_2 t))$
	Log-logistic: $\alpha_0/(1 + \alpha_1 \exp(-\alpha_2 \ln(t))) + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= 1/(1 + \alpha_1 \exp(-\alpha_2 \ln(t)))$
$\partial\phi/\partial\alpha_1$	$= [\alpha_0 \exp(-\alpha_2 \ln(t))][1 + \alpha_1 \exp(-\alpha_2 \ln(t))]^2$
$\partial\phi/\partial\alpha_2$	$= [\alpha_0\alpha_1 \ln(t) \exp(-\alpha_2 \ln(t))]/ (1 + \alpha_1 \exp(-\alpha_2 \ln(t)))^2$
	Richard's: $\phi(t) = \alpha_0/\left[\left(1+\alpha_1 \exp(-\alpha_2 t))\right)^{\frac{1}{\alpha_3}}\right] + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= \left(1+\alpha_1 \exp(-\alpha_2 t)\right)^{\frac{-1}{\alpha_3}}$
$\partial\phi/\partial\alpha_1$	$= \left(-\alpha_0/\alpha_3\right)\left(1+\alpha_1 \exp(-\alpha_2 t)\right)^{\frac{-1}{\alpha_3}-1}\left(\exp(-\alpha_2 t)\right)$
$\partial\phi/\partial\alpha_2$	$= \left(\alpha_0\alpha_1 t/\alpha_3\right)\left(1+\alpha_1 \exp(-\alpha_2 t)\right)^{\frac{-1}{\alpha_3}-1}\left(\exp(-\alpha_2 t)\right)$
$\partial\phi/\partial\alpha_3$	$= \alpha_0\left(1+\alpha_1 \exp(-\alpha_2 t)\right)^{\frac{-1}{\alpha_3}} \ln\left(1+\alpha_1 \exp(-\alpha_2 t)\right)\alpha_3^{-2}$
	Weibull: $\phi(t) = \alpha_0 - \alpha_1 \exp(-\alpha_2 t^{\alpha_3}) + \varepsilon$
$\partial\phi/\partial\alpha_0$	$= 1.0$
$\partial\phi/\partial\alpha_1$	$= -\exp\left(-\alpha_2 t^{\alpha_3}\right)$
$\partial\phi/\partial\alpha_2$	$= \exp\left(-\alpha_2 t^{\alpha_3}\right)\left(\alpha_1 t^{\alpha_3}\right)$
$\partial\phi/\partial\alpha_3$	$= \exp\left(-\alpha_2 t^{\alpha_3}\right)\alpha_1\alpha_2 \ln(t) t^{\alpha_3}$

Table 5: Partial derivatives of the Schnute, Morgan-Mercer-Flodin, Champan-Richard and Stannard growth models

	Schnute: $\phi(t)=\left(\alpha_0+\alpha_1\exp(\alpha_2 t)\right)^{\alpha_3}+\varepsilon$
$\partial\phi/\partial\alpha_0$	$=\left(\alpha_3\left(\alpha_0+\alpha_1\exp(\alpha_2 t)\right)^{\alpha_3-1}\right)$
$\partial\phi/\partial\alpha_1$	$=\left(\alpha_3\exp(\alpha_2 t)\right)\left(\alpha_0+\alpha_1\exp(\alpha_2 t)\right)^{\alpha_3-1}$
$\partial\phi/\partial\alpha_2$	$=\left(\alpha_1\alpha_3 t\exp(\alpha_2 t)\right)\left(\alpha_0+\alpha_1\exp(\alpha_2 t)\right)^{\alpha_3-1}$
$\partial\phi/\partial\alpha_3$	$=\left(\alpha_0+\alpha_1\exp(\alpha_2 t)\right)^{\alpha_3}\ln\left(\alpha_0+\alpha_1\exp(\alpha_2 t)\right)$
	Morgan-Mercer-Flodin: $\phi(t)=\alpha_0-(\alpha_0-\alpha_1)/(1+(\alpha_2 t)^{\alpha_3})+\varepsilon$
$\partial\phi/\partial\alpha_0$	$=1-(1+(\alpha_2 t)^{\alpha_3}$
$\partial\phi/\partial\alpha_1$	$=(1+(\alpha_2 t)^{-\alpha_3}$
$\partial\phi/\partial\alpha_2$	$=\left(\alpha_3(\alpha_0-\alpha_1)(\alpha_3 t)^{\alpha_3}\right)/\left(\alpha_2((1+(\alpha_2 t)^{\alpha_3})^2\right)$
$\partial\phi/\partial\alpha_3$	$=\left((\alpha_0-\alpha_1)\ln(\alpha_2 t)(\alpha_2 t)^{\alpha_3}\right)/\alpha_2\left(1+(\alpha_2 t)^{\alpha_3}\right)^2$
	Chapman-Richard: $\phi(t)=\alpha_0(1-\alpha_1\exp(-\alpha_2 t))^{\frac{1}{1-\alpha_3}}+\varepsilon$
$\partial\phi/\partial\alpha_0$	$=\left(1-\alpha_1\exp(-\alpha_2 t)\right)^{\frac{1}{1-\alpha_3}}$
$\partial\phi/\partial\alpha_1$	$=\left(-\alpha_0/(1-\alpha_3)\right)\left(1-\alpha_1\exp(-\alpha_2 t)\right)^{\frac{1}{1-\alpha_3}-1}\left(\exp(-\alpha_2 t)\right)$
$\partial\phi/\partial\alpha_2$	$=\left(\alpha_0\alpha_1 t/(1-\alpha_3)\right)\left(1-\alpha_1\exp(-\alpha_2 t)\right)^{\frac{1}{1-\alpha_3}-1}\left(\exp(-\alpha_2 t)\right)$
$\partial\phi/\partial\alpha_3$	$=\left(\alpha_0/(1-\alpha_3)^2\right)\left(1-\alpha_1\exp(-\alpha_2 t)\right)^{\frac{1}{1-\alpha_3}}\ln\left(1-\alpha_1\exp(-\alpha_2 t)\right)$
	Stannard: $\phi(t)=\alpha_0\left[1+\exp\left(-((\alpha_1+\alpha_2 t)/\alpha_3)\right)\right]^{\alpha_3}$
$\partial\phi/\partial\alpha_0$	$=\left(1+\exp\left(-((\alpha_1+\alpha_2 t)/\alpha_3)\right)\right)^{-\alpha_3}$
$\partial\phi/\partial\alpha_1$	$=\alpha_0\exp\left(-(\alpha_1+\alpha_2 t/\alpha_3)\right)\left(1+\exp\left(-(\alpha_1+\alpha_2 t/\alpha_3)\right)\right)^{1-\alpha_3}$
$\partial\phi/\partial\alpha_2$	$=\alpha_0 t\exp\left(-((\alpha_1+\alpha_2)/\alpha_3)\right)\left(1+\exp\left(-((\alpha_1+\alpha_2)/\alpha_3)\right)\right)^{1-\alpha_3}$
$\partial\phi/\partial\alpha_3$	$=\begin{array}{l}\alpha_0\left[1+\exp\left(-((\alpha_1+\alpha_2 t)/\alpha_3)\right)\right]^{-\alpha_3}\ln\left[1+\exp\left(-((\alpha_1+\alpha_2 t)/\alpha_3)\right)\right]\\ *\left\{-\left(((\alpha_1+\alpha_2 t)\exp\left(-((\alpha_1+\alpha_2 t)/\alpha_3)\right)\right)/\alpha_3\right)+\left((\alpha_1+\alpha_2 t)/\alpha_3\right)\right\}\end{array}$

simply use the F statistic to obtain conclusions at any stated level of significant[7-8]. This study considers several procedures to test the goodness of fit for nonlinear model, such as confidence interval of the parameters estimated; asymptotic correlation matrix, residual analysis and normality probability plot were carried out. The Mean Squares Error (MSE), Mean Absolute Error (MAE), correlation coefficient between actual values and estimated values and the Mean Absolute Percentage Error (MAPE) were used to measure the model performance.

Partial derivatives of the models: Let the symbols of the parametersα, β, κ and δ, in the nonlinear model be replaced by new symbols α_0, α_1, α_2 and α_3 respectively. The parameters for all models considered here are defined as follows: α_0 is the asymptote or the potential maximum of the response variable; α_1 is the biological constant; α_2 is the parameter governing the rate at which the response variable approaches its potential maximum; and α_3 is the allometric constant. The partial derivatives of the models with respect to each parameter ($\partial\phi/\partial\alpha_j$) are given in Table 3 to 5. The NLIN procedure in[10] requires that the integral forms and the partial derivatives of the nonlinear models must be entered in the program using valid SAS syntax.

The Marquardt algorithm requires that an initial value for each parameter be estimated. Initial value specification is one of the most difficult problems encountered in estimating parameters of nonlinear models [7]. In appropriate initial values will result in longer iteration, greater execution time, non-convergence of the iteration and possibly convergence to unwanted local minimum sum of squares residual. The simplest parameter to specify is the α_0. This is attributed to the clarity of its definition. The parameter α_0 is defined as maximum possible value of the dependent variable determined by the productive capacity of the experimental site. Therefore, in our case α_0 was specified as the maximum value of the response variable in the data. Then, the α_2 parameter is defined as the rate constant at which the response variable approaches its maximum possible value α_0. For modeling biological growth variables the, allometric constant α_3 lies between zero and one for the Chapman-Richards growth model and is positive for the von Bertalanffy, Richard's, Weibull and Morgan-Mercer-Flodin growth models. Finally, α_1 parameter can be specified by evaluate the models at the start of growth when the predictor variable is zero.

RESULTS AND DISCUSSION

The least squares estimates of the parameters of the nonlinear models for oil palm yield-age relationship are given in Table 6 and 7. The parameter estimates for the logistic, Gompertz, negative exponential, monomolecular and Morgan-Mercer-Flodin growth functions are all statistically significant at the 5% level. Estimates of α_1 and α_3 for von Bertalanffy, Richard's and Chapman-Richard growths model are not statistically significant at the 5% level. Parameter estimates of the Weibull and Stannard growth models except α_1 are statistically significant at 5% level. The Marquardt iteration procedure was converge for all growth models with various numbers of iterations. The minimum iteration is 8 for negative exponential growth model and Chapman-Richard growth model recorded the highest iteration i.e. 43. The Schnute model is not presented as it failed to converge.

Statistical significance of the parameters of the nonlinear models was determined by evaluating the 95% asymptotic confidence intervals of the estimated parameters. The null hypothesis H_0: $\alpha_j = 0$ was rejected when the 95% asymptotic confidence interval of α_j does not include zero. The 95% asymptotic confidence intervals for each growth model are presented at last column Table 6 and 7.

Table 8 presents the asymptotic correlation coefficients among the parameter estimated. All asymptotic correlation coefficients are relatively small, except for von Bertalanffy {(α_1, α_2)=0.9248; (α_1, α_3)=0.9970; (α_2, α_3)=0.9496}, Richard's {(α_1, α_2) =0.9752; (α_1, α_3) =0.9937; (α_2, α_3)=0.9495} and Weibull {(α_0, α_3)=-0.9999; (α_1, α_2)=-0.9475} respectively.

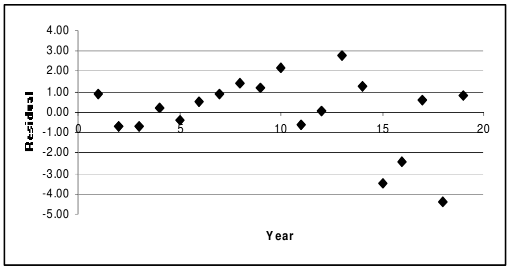
Table 9 provides predicted values of fresh fruit bunch over the range of age using the least squares parameter estimates derived from the Marquardt algorithm. The logistic, von Bertalanffy, Richard's and Stannard models have produced a slightly smaller mean absolute percentage error (0.03) compared to the Gompertz and MMF (0.04), the negative exponential, monomolecular and Weibull (0.05), the log-logistic (0.06) and Chapman-Richards (0.07). All the models in Table 9 appear to predict reasonable estimates over the entire range of age. Figure 2 shows the plotted residual

Table 6: Parameter estimates of the logistic, Gompertz, negative exponential, monomolecular, log-logistic, Richard's and Weibull growth models for yield-age relationship

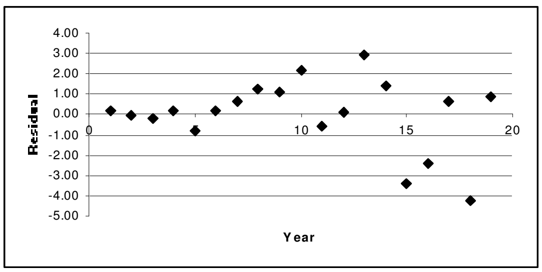
Model parameter	Parameter Estimated	Asymptotic standard error	Asymptotic confidence interval lower	upper
Logistic				
α_0	37.0806	0.5327	35.9514	38.2098
α_1	4.8149	1.3115	2.0345	7.5952
α_2	0.7817	0.1087	0.5511	1.0122
Gompertz				
α_0	37.1788	0.5701	35.9703	38.3874
α_1	2.2683	0.4265	1.3642	3.1724
α_2	0.6132	0.0854	0.4321	0.7943
Negative Exponential				
α_0	37.5017	0.6643	36.1001	38.9033
α_2	0.4046	0.0362	0.3282	0.4811
Monomolecular				
α_0	37.3235	0.6565	35.9317	38.7151
α_1	1.1408	0.1367	0.8511	1.4305
α_2	0.4592	0.0689	0.3130	0.6055
Log-Logistic				
α_0	38.1172	1.0667	35.8559	40.3785
α_1	3.1947	0.8678	1.3549	5.0344
α_2	1.8874	0.3245	1.1995	2.5754
Richard's				
α_0	37.0418	0.5698	37.0418	38.2564
α_1	11.0433	33.6452	-60.6695	82.7561
α_2	0.8729	0.4059	0.0076	1.7383
α_3	1.5205	2.0391	-2.8257	5.8667
Weibull				
α_0	37.3234	0.8887	35.4291	39.2178
α_1	-5.2452	8.9982	-24.4245	13.9339
α_2	0.3415	0.0906	0.1483	0.5347
α_3	1.3442	0.0014	1.3411	1.3472

Table 7: Parameter estimates of the MMF, von Bertalanffy, Chapman-Richard and Stannard growth models for yield-age relationship

Model parameter	Parameter Estimated	Asymptotic standard error	Asymptotic confidence interval lower	upper
Morgan-Mercer-Flodin				
α_0	37.2032	0.6724	35.7700	38.6365
α_1	11.5236	2.5198	6.1525	16.8943
α_2	0.3534	0.0355	0.2776	0.4292
α_3	3.4347	0.8877	1.5425	5.3270
Von Bertalanffy				
α_0	37.0416	0.5698	35.8270	38.2562
α_1	-0.0455	0.1979	-0.4673	0.3763
α_2	0.8731	0.4058	0.0080	1.7382
α_3	2.5203	2.0388	-1.8254	6.8661
Chapman-Richard (without initial stage)				
α_0	35.8502	0.8162	34.1106	37.5898
α_1	0.4927	2.3322	-4.4783	5.4637
α_2	0.4488	0.1449	0.1397	0.7579
α_3	0.6155	2.8893	-5.5430	6.7740
Stannard				
α	37.0415	0.56598	35.8269	38.2561
α_1	-1.5799	0.2544	-2.1222	-1.0376
α_2	0.5743	0.5236	-0.5417	1.6904
α_3	0.6577	0.8825	-1.2232	2.5388



Logistic model



Gompertz model

Fig. 1: Residual plot for Logistic and Gompertz growth models

Table 8: Asymptotic correlation for each nonlinear growth models fitted

Model	Asymptotic correlation
Logistic	$(\alpha_0, \alpha_1) = -0.1743$; $(\alpha_0, \alpha_2) = -0.3631$; $(\alpha_1, \alpha_2) = 0.8863$
Gompertz	$(\alpha_0, \alpha_1) = -0.2324$; $(\alpha_0, \alpha_2) = -0.4398$; $(\alpha_1, \alpha_2) = 0.8675$
Von Bertalanffy	$(\alpha_0, \alpha_1) = -0.2911$; $(\alpha_0, \alpha_2) = -0.3891$; $(\alpha_0, \alpha_3) = -0.3073$; $(\alpha_1, \alpha_2) = 0.9248$; $(\alpha_1, \alpha_3) = 0.9970$; $(\alpha_2, \alpha_3) = 0.9496$.
Negative exponential	$(\alpha_0, \alpha_2) = -0.5911$
Monomolecular	$(\alpha_0, \alpha_1) = -0.3532$; $(\alpha_0, \alpha_2) = -0.5552$; $(\alpha_1, \alpha_2) = 0.8536$
Log-logistic	$(\alpha_0, \alpha_1) = -0.3162$; $(\alpha_0, \alpha_2) = -0.7245$; $(\alpha_1, \alpha_2) = 0.7799$
Richard's	$(\alpha_0, \alpha_1) = -0.3212$; $(\alpha_0, \alpha_2) = -0.3892$; $(\alpha_0, \alpha_3) = -0.3072$; $(\alpha_1, \alpha_2) = 0.9752$; $(\alpha_1, \alpha_3) = 0.9937$; $(\alpha_2, \alpha_3) = 0.9495$.
Weibull	$(\alpha_0, \alpha_1) = -0.3766$; $(\alpha_0, \alpha_2) = 0.2781$; $(\alpha_0, \alpha_3) = -0.9999$; $(\alpha_1, \alpha_2) = -0.9475$; $(\alpha_1, \alpha_3) = 0.3763$; $(\alpha_2, \alpha_3) = -0.2778$.
Morgan-	$(\alpha_0, \alpha_1) = -0.2426$; $(\alpha_0, \alpha_2) = -0.0015$; $(\alpha_0, \alpha_3) = -$
0.5212;	
Mercer-Flodin	$(\alpha_1, \alpha_2) = -0.7558$; $(\alpha_1, \alpha_3) = 0.6085$; $(\alpha_2, \alpha_3) = -0.5218$.
Chapman-Richard	$(\alpha_0, \alpha_1) = -0.7459$; $(\alpha_0, \alpha_2) = 0.4445$; $(\alpha_0, \alpha_3) = -0.6289$; $(\alpha_1, \alpha_2) = -0.6471$; $(\alpha_1, \alpha_3) = 0.9104$; $(\alpha_2, \alpha_3) = -0.2844$.
Stannard	$(\alpha_0, \alpha_1) = -0.0364$; $(\alpha_0, \alpha_2) = 0.2542$; $(\alpha_0, \alpha_3) = 0.3077$; $(\alpha_1, \alpha_2) = -0.6204$; $(\alpha_1, \alpha_3) = -0.5031$; $(\alpha_2, \alpha_3) = 0.9871$.

of fitted nonlinear models. The plot showed that the residual are distributed along the zero line and we can conclude that the residual from the fitted models are normally distributed.

Next, the models were diagnosed using error analysis. The error analysis is performed to analyze the difference between the error values and the estimated values of observation. This analysis is able to investigate the goodness of fit of the nonlinear models graphically and some of the plots are illustrated in Fig. 1. The scattered plot of the errors is important in deciding whether the residual values are uniformly distributed, there is no systematic trend of the residual values or the variance is consistent or not. If the error plot showed that the errors have a homogeneous variance then the model are adequate to model the data. The figure shows that the errors are scattered uniformly and the residual variance is homogeneous. It is proved that the nonlinear growth models used in this study has the ability concerning the behaviors of the oil palm growth data.

When nonlinear models are fitted to a biological growth data set statistical non-significance of the estimated parameters might imply one of the following:

- One or more parameters in the model may not be useful, or more accurately, a reparameterized model involving fewer parameters might be more appropriate;
- The biological growth data used for fitting the model are not adequate for estimating all the parameters; or
- The model assumptions do not conform with the biological system being modeled.

The argument in (ii) was the case with the von Bertalanffy and the Chapman-Richard growth models. Investigation of the differential forms and second derivatives of the von Bertalanffy and the Chapman-Richard models indicate that the functions are suitable to model a system that encompasses the entire range of cycle (i.e. juvenile, adolescent, mature and senescent stages) of the biological response variable. However, the FFB growth measurements considered in this study lacks data on juvenile stages of growth. Hence, non-significance of two of the parameters of the two models might be attributed to this cause. To support this argument we have included an initial data point (age = 0, FFB = 0) to the data and refitted the von Bertalanffy and the Chapman-Richard models. Table 10 shows the parameter estimates, asymptotic standard error and asymptotic 95% confidence intervals for each parameter of these two models.

Without inclusion of the initial data point two of the parameters (α_1 and α_3) are not statistically significant (Table 7). However, inclusion of the initial data point has resulted in only the Chapman-Richard growth model showing statistically significant estimates of the three parameters. Meanwhile, the von Bertalanffy growth model did not shows any improvement. Inclusion of any additional data point from an early stage of growth will result in significant improvement in the estimate of the parameters of the Chapman-Richard model. Table 9 also indicated that with initial values, the MAPE was reduced from 0.07 to 0.05. This illustrates that significance of the parameters of the Chapman-Richard growth model depends on the range of the growth data.

Table 9: Actual and predicted values of FFB yield, the associated measurement error and correlation coefficient between actual and predicted

Year	FFB yield	Logistic	Gompertz	Von Ber-alanffy	Negative Exponential	Mono-molecular	Log Logistic	Richard	Weibull	MMF	Chapman-Richard	Richard*	Chapman-Stannard	
0	0	-	-	-	-	-	-	-	-	-		-	0.92	-
1	11.78	11.58	10.88	11.91	12.48	10.42	9.09	11.91	10.43	12.23	13.43	10.05	11.91	
2	18.43	18.46	19.11	18.28	20.81	20.33	20.46	18.28	20.33	17.51	20.01	19.51	18.28	
3	25.21	25.37	25.93	25.12	26.36	26.59	27.19	25.12	26.59	25.65	25.09	26.33	25.12	
4	30.78	30.62	30.59	30.61	30.07	30.54	30.90	30.61	30.54	31.21	28.71	30.71	30.61	
5	33.03	33.81	33.45	33.98	32.54	33.04	33.05	33.98	33.04	34.02	31.18	33.38	33.98	
6	35.66	35.51	35.11	35.68	34.19	34.62	34.38	35.68	34.61	35.40	32.82	34.97	35.68	
7	36.96	36.35	36.04	36.46	35.29	35.61	35.26	36.46	35.61	36.11	33.90	35.91	36.46	
8	37.97	36.74	36.56	36.79	36.03	36.24	35.86	36.79	36.24	36.50	34.60	36.45	36.79	
9	38.04	36.92	36.84	36.94	36.52	36.64	36.28	36.94	36.64	36.73	35.05	36.77	36.94	
10	39.20	37.01	37.00	37.00	36.85	36.89	36.60	37.00	36.89	36.87	35.34	36.95	37.00	
11	36.50	37.05	37.08	37.02	37.06	37.05	36.84	37.02	37.05	36.96	35.52	37.06	37.02	
12	37.21	37.07	37.13	37.03	37.21	37.15	37.03	37.03	37.15	37.02	35.64	37.12	37.03	
13	39.97	37.07	37.15	37.04	37.31	37.21	37.18	37.04	37.21	37.07	35.72	37.16	37.04	
14	38.45	37.08	37.16	37.04	37.37	37.25	37.30	37.04	37.25	37.10	35.76	37.18	37.04	
15	33.65	37.08	37.17	37.04	37.42	37.28	37.40	37.04	37.28	37.12	35.80	37.19	37.04	
16	34.71	37.08	37.17	37.04	37.44	37.30	37.48	37.04	37.30	37.14	35.82	37.19	37.04	
17	37.75	37.08	37.18	37.04	37.46	37.31	37.55	37.04	37.31	37.15	35.83	37.20	37.04	
18	32.81	37.08	37.18	37.04	37.48	37.31	37.60	37.04	37.31	37.16	35.84	37.20	37.04	
19	37.99	37.08	37.18	37.04	37.48	37.32	37.65	37.04	37.32	37.17	35.84	37.20	37.04	
MSE		2.96	3.15	2.94	4.06	3.72	4.64	2.94	3.72	3.20	6.19	3.41	2.94	
MAE		1.22	1.35	1.22	1.61	1.53	1.72	1.22	1.53	1.37	2.27	1.45	1.22	
MAPE		0.03	0.04	0.03	0.05	0.05	0.06	0.03	0.05	0.04	0.07	0.05	0.03	
r		0.97	0.97	0.97	0.96	0.96	0.96	0.97	0.96	0.97	0.96	0.97	0.97	

* With

Table 10: he parameter estimates an asymptotic correlation for von Bertalanffy and Chapman-Richard when an initial growth response data point is added

Model parameter	Parameter Estimated	Asymptotic standard error	Asymptotic confidence interval lower	upper
Von Bertalanffy				
α_0	37.2017	0.6140	39.9001	38. 5033
α_1	5.5385	9.5941	-14.7999	25.8769
α_2	0.5498	0.1238	0.2873	0.8124
α_3	0.4826	0.3959	-0.3566	1.3218
Chapman-Richard				
α_0	37.2036	0.6256	35.8773	38.5298
α_1	0.8530	0.3581	0.0937	1.6122
α_2	0.5498	0.1265	0.2816	0.8181
α_3	0.4822	0.1018	1.3340	
Model	Asymptotic correlation			
Von Bertalanffy	$(\alpha_0, \alpha_1) = 0.1127$; $(\alpha_0, \alpha_2) = -0.4229$; $(\alpha_0, \alpha_3) = -0.2532$; $(\alpha_1, \alpha_2) = -0.8174$; $(\alpha_1, \alpha_3) = -0.9887$; $(\alpha_2, \alpha_3) = 0.8428$			
Chapman-Richard	$(\alpha_0, \alpha_1) = 0.2264$; $(\alpha_0, \alpha_2) = -0.4879$; $(\alpha_0, \alpha_3) = -0.2988$; $(\alpha_1, \alpha_2) = -0.6940$; $(\alpha_1, \alpha_3) = -0.9571$; $(\alpha_2, \alpha_3) = 0.8516$.			

This study provide the statistical requirement for estimating parameters of nonlinear growth models, statistical testing for the parameters estimated and interpret some of the relevant statistical output from oil palm perspective. The NLIN procedure in SAS does not guarantee that the iteration converges to a global minimum sum of squares residual[10]. Hence, an alternative approach for avoiding the problem of non-convergence or convergence to u nwanted local minimum sum of squares residual is to specify a grid values for each parameter. Then NLIN evaluates the residual of sum squares errors at each combination of values to determine the best initial values for the iteration process. Initial values may be intelligent guesses or preliminary estimates based on available information. Initial values may, for example, be values suggested by the information gained in fitting a similar equation in a different place or values suggested as ‘about right’ by the experimenter based on personal experience and knowledge. Based on meaningful biologial definitions of the parameters of the nonlinear models, expression to specify initial values for the asymptote and the biological constant were developed. These expressions were found useful to specify initial values of the parameters for modeling the sample fresh fruit bunches data used in the stud.

Table 11: The summary of number of iteration and sum of squares for each model

Model	Number of iterative	Sum of squares
Logistic	20	56.1502
Gompertz	22	59.9054
Von Bertalanffy	36	55.7957
Negative exponential	8	77.0909
Monomolecular	26	70.6852
Log logistics	22	88.2415
Richard’s	26	55.7958
Weibull	18	70.6852
Morgan-Mercer-Flodin	21	60.8861
Chapman-Richards	34	117.6159
Chapman-Richards (initial stage)	43	65.6814
Stannard	42	55.7957

CONCLUSION

Some of the models such as the negative exponential and monomolecular have no point of inflection and are not sigmoid shape. Hence, the models are not appropriate for modeling the entire range of the life cycle of biological response variables such as oil palm yield growth that exhibit a sigmoid pattern over time (reason (iii)) in previous part. This study found that the Gompertz, logistic, log-logistic, Morgan-Mercer-Flodin and Chapman-Richard growth models have the ability and suitable for quantifying a growth phenomenon that exhibits a sigmoid pattern over time. Base on the statistical testing and sum of squares error (Table 11), the model in the first rank is the Logistic model, second rank is the Gompertz model and followed by Morgan-Mercer-Flodin, Chapman-Richard (with initial stage) and the Log-logistic growth model.

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