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PROBABILISTIC PERIODIC REVIEW <Q_M, N> INVENTORYMODELUSING LAGRANGE TECHNIQUE AND FUZZY ADAPTIVE PARTICLE SWARM OPTIMIZATION

[1]Hollah, O.M., [2]N.A. El-Hefnawy and [3]H.A. Fergany

[1]Department of Mathematics, Faculty of Science, Tanta University, Egypt
[2]Depterment of OR, Faculty of Computers and Information, Shiben El-Kom, Menoufia University, Egypt
[3]Department of Mathematics, High Institute for Computers, Information and Management, Tanta, Egypt

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ABSTRACT

The integration between inventory model and Artificial Intelligent (AI) represents the rich area of research since last decade. In this study we investigate probabilistic periodic review <Q_m, N> inventory model with mixture shortage (backorder and lost sales) using Lagrange multiplier technique and Fuzzy Adaptive Particle Swarm Optimization (FAPSO) under restrictions. The objective of these algorithms is to find the optimal review period and optimal maximum inventory level which will minimize the expected annual total cost under constraints. Furthermore, a numerical example is applied and the experimental results for both approaches are reported to illustrate the effectiveness of overcoming the premature convergence and of improving the capabilities of searching to find the optimal results in almost all distributions.

Keywords: Inventory System, Periodic Review Model, Particle Swarm Optimization, Fuzzy Adaptive Particle Swarm Optimization

1. INTRODUCTION

In some cases while a few customers are ready to wait till the next arrival of stock (backorder case), the remaining may be impatient and would persist on satisfying their demand immediately from some other sources (lost sales case). Inventory models which involve both backorders and lost sales are known as models with a mixture shortage. First solution to such a model was derived by (Montgomery *et al.*, 1973). A similar model for variable lead time with fixed reorder point was analyzed by (Quyang and Wu, 1996). Hariga and Ben Daya (1999) discussed both periodic and continuous review models with a mixture of backorders and lost sales in case of full and partial demand information. Abuo-El-Ata *et al.* (2002) studied probabilistic multi-item Inventory model with varying order cost under two restrictions. Fergany (2005) described periodic review model with zero lead time under constraints and varying order cost. Fergany and Elwakeel (2006) introduced constrained probabilistic lost sales inventory system with normal distribution and varying order cost.

After appear the shortcomings of the traditional methods to deal with complexities of nonlinear programming with low and high dimensions, Particle Swarm Optimization (PSO) appears which has exhibited good performance for solving problems in wide range of applications such as in engineering design and computer science whereas the PSO is easy to implement in computer simulations. The PSO has fewer operators to adjust in the implementation and it can be flexibly combined with other optimization techniques to build a hybrid algorithm, the mechanism of PSO facilitates has better convergence performance than some other optimization procedures.

A new optimizer using particle swarm theory derived by (Eberhart and Kennedy, 1995). Tasgetiren and Liang (2003) presents a binary particle swarm optimization algorithm for lot sizing problem. Andries (2005)

Corresponding Author: Fergany, H.A., Department of Mathematics, Faculty of Science, Tanta University, Egypt

discussed the fundamentals of computational swarm intelligence. Kang *et al*. (2006) studied a novel Fuzzy Adaptive Strategy Optimization (FAPSO) for the particle swarm algorithm. Xiaobin *et al*. (2007) described fuzzy economic order quantity inventory models without backordering. Parsopiulos *et al*. (2008) investigate particle swarm optimization for tacking continuous review inventory models. Kannan *et al*. (2009) introduced analysis of closed loop supply chain using genetic algorithm and PSO. Taleizadeh *et al*. (2010) studied a particle swarm optimization approach for constraint joint single buyer-single vendor inventory problem with changeable lead time and (r, Q) policy in supply chain. Hiremath *et al*. (2010) discussed optimization for efficient supply chain using swarm intelligence: An Outsourcing Perspective. Piperagkas *et al*. (2011) applying PSO and DE on multi-item inventory problem with supplier selection.

In this study, we investigate a probabilistic Single-Item Single-Source (SISS) inventory model with varying mixture shortage cost under two restrictions, which one is on the expected backorder cost and the other is on the expected lost sales cost. The optimal maximum inventory level Q_m^*, the optimal time between reviews N^* and the minimum Expected Total Cost (min E (TC)) are obtained, some special cases are deduced, some distributions are implemented and results of numerical computations for optimum parameters of this model using Lagrange multiplier technique and fuzzy adaptive particle swarm optimization and their comparisons are presented.

The rest of this study is organized as follows: Section 2 presents constrained probabilistic single item periodic review $\langle Q_m, N\rangle$ model with Varying Mixture Shortage. Section 3 presents Standard Particle Swarm Optimization (SPSO). Section 4 the solution procedure of FAPSO is proposed. Section 5 presents experiments and results of numerical example to test the validity and performance of the approach. Section 6 presents a comparative study between two approaches. Section 7 concludes the study and future work.

2. CONSTRAINED PROBABILISTIC SINGLE ITEM PERIODIC REVIEW $\langle Q_M, N\rangle$ MODEL WITH VARYING MIXTURE SHORTAGE

The following assumptions are made for developing the model:

- The time between the arrivals of two successive orders rather than between the placements of two successive orders is called a period
- N is the time between reviews
- A sufficient quantity is ordered to bring the inventory level up to level Q_m, where Q_m is the maximum inventory level
- The varying backorder cost for the item per period is C_bN^{β}, where C_b is the backorder cost per period, the varying lost sales cost for the item per period is $C_L N^{\beta}$, where C_L is the lost sales cost per period and the cost is independent of the length of time for which the backorder and lost sales exists and β are constant real numbers selected to provide the best fit of estimated expected cost function
- The behavior of the periodic review system with partial backorders and lost sales case shown in **Fig. 1**

The expected annual total cost is the sum of the expected review cost, expected ordering cost, expected holding cost, expected backorder cost and the expected lost sales cost respectively Equation 1:

$$E(TC) = E(RC) + E(OC) + E(HC) + E(BC) + E(LC) \qquad (1)$$

Where:

$$E(RC) = \frac{C_r}{N}, C_r \text{ the cost of making making a review}$$

$$E(OC) = \frac{C_o}{N},$$

C_o the cost of placing an order (ordering cost) per period

The expected number of backorders incurred per year E (BC) is the expected number of backorders incurred per period multiplied by the number of orders per year.

Consider the first case where L is constant (where L the lead time between the placement of an order and its receipt). The expected number of backorders incurred per period is $\int_{Q_m}^{\infty}(x-Q_m)\, f(x; L+N)dx$, where x is the demand between the time an order is placed and the time this order arrives (the demand during lead time), f (x; $L+N$) is the probability density function of the lead time demand x during the time interval of length $L+N$.

Suppose now that the lead time L is a random variable with density $g(L)$. Let L_{min}, L_{max} be the lower and upper limits respectively to the possible range of the lead time values. If L_1, L_2 are the lead times for the orders placed at times t and $t+N$ respectively, then the expected number of backorders incurred per period is given by:

$$\bar{S}(Q_m) = \int_{L_{\min}}^{L_{\max}}\int_{L_{\min}}^{L_{\max}}\int_{Q_m}^{\infty}(x-Q_m)\, f(x; L_2+N)$$

$$g(L_2)g(L_1)dxdL_2\,dL_1 = \int_{Q_m}^{\infty}(x-Q_m)h(x;N)dx$$

Where:

$$\int_{L_{\min}}^{L_{\max}} g(L_1)dL_1 = 1, h(x;N)$$
$$= \int_{L_{\min}}^{L_{\max}} f(x;L_2+N)g(L_2)dL_2$$

It is necessary to known that $L_{max} < L_{min} + N$, hence the expected backorder cost incurred per year Equation 2 and 3:

$$E(BC) = C_b \gamma N^{\beta-1}\overline{S}(Q_m)$$
$$= C_b \gamma N^{\beta-1} V * \int_{Qm}^{\infty}(x-Q_m)h(x;N)d \quad (2)$$

$$E(LC) = C_L(1-\gamma)N^{\beta-1}\overline{S}(Q_m)$$
$$= C_L(1-\gamma)N^{\beta-1}\int_{Qm}^{\infty}(x-Q_m)h(x;N)dx \quad (3)$$

where, γ is a fraction of unsatisfied demand that will be backorder while the remaining fraction (1-γ) is completely lost.

$E(HC) = \frac{C_h I}{N}$, C_h is the holding (carrying) cost of the item per period and $\overline{I}$ the average inventory level per period. Let ε (x; Q_m) be the on hand inventory when the procurement arrives. If the lead time demand is x and the maximum inventory level immediately after the arrival of the procurement is Q_m, then:

$$\varepsilon(x;Q_m) = \begin{cases} Q_m - x & Q_m - x \ge 0 \\ 0 & Q_m - x < 0 \end{cases}$$

And the expected on hand inventory when the procurement arrives (safety stock) is:

$$= \int_0^{Qm} \varepsilon(x;Q_m)h(x;N)dx = \int_0^{Qm}(Q_m - x)h(x;N)dx$$
$$= Q_m - \mu + \int_{Qm}^{\infty}(x-Q_m)h(x;N)dx$$

μ is the expected lead time demand.

Hence if the expected on hand inventory immediately after the arrival of a procurement is S, it is therefore S-DN gust prior to the arrival of a procurement in the next period. Thus the expected on hand inventory varies between S and S-DN. where DN is the expected demand during the time N between reviews and D is the average demand rate. Then the average inventory level per period is given by:

$\overline{I}$ = (the expected inventory level (at eh beginning of an inventory periodat the end of the period)/2:

$$\overline{I} = N\left[Q_m - \mu - \frac{DN}{2} + (1-\gamma)\int_{Qm}^{\infty}(x-Q_m)h(x;N)dx\right]$$

Which yields Equation 4:

$$E(HC) = C_h\left(Q_m - \mu - \frac{DN}{2} + (1-\gamma)\int_{Qm}^{\infty}(x-Q_m)h(x;N)dx\right)$$
$$= C_h\left(Q_m - \mu - \frac{DN}{2} + (1-\gamma)\overline{S}(Q_m)\right) \quad (4)$$

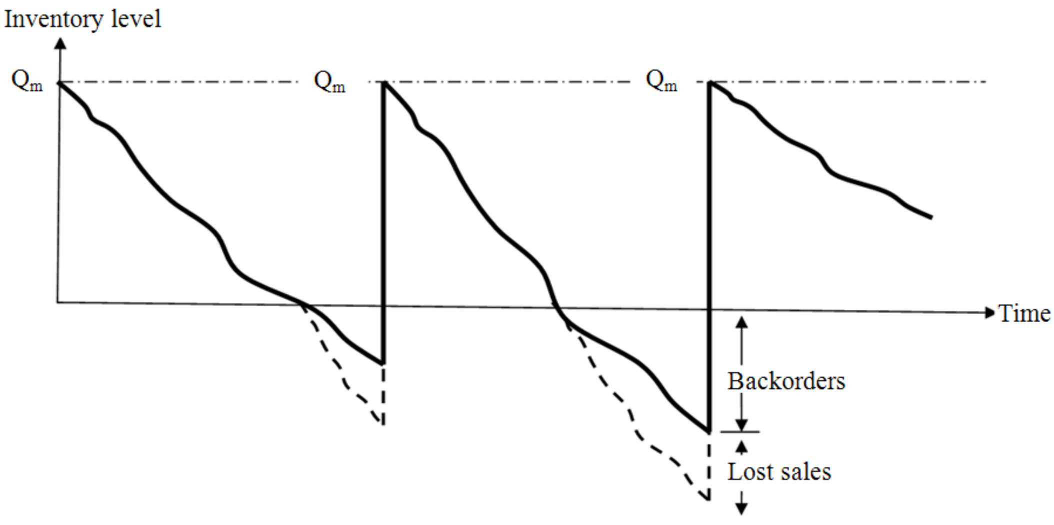

Fig. 1. The behavior of the periodic review system with partial backorders and lost sales case

Then the expected annual total cost is given by:

$$E(TC(Q_m,N))=\frac{C_r}{N}+\frac{C_o}{N}+C_h\left(Q_m-\mu-\frac{DN}{2}\right)$$
$$+\left[C_b\gamma N^{\beta-1}+\left(C_L N^{\beta-1}+C_h\right)(1-\gamma)\right]*$$
$$\int_{Q_m}^{\infty}(x-Q_m)h(x;N)dx \quad (5)$$
$$=\frac{C_r}{N}+\frac{C_o}{N}+C_h(Q_m-\mu-\frac{DN}{2})$$
$$+\left[C_b\gamma N^{\beta-1}+(C_L N^{\beta-1}+C_h)(1-\gamma)\right]\bar{S}(Q_m)$$

The objective is to determine the optimal values Q_m^*, N^* that minimize the expected annual total cost $E(TC(Q_m, N))$ under the following constraints:

$$E(BC)\le K_b \quad E(LC)\le K_L \quad (6)$$

To solve this primal function which is a convex programming problem, Equation 5 and 6 can be written in the following form:

$$Min\left(E\left(TC\left(Q_m,N\right)\right)\right)$$
$$=\frac{C_r}{N}+\frac{C_o}{N}+C_h\left(Q_m-\mu-\frac{DN}{2}\right)$$
$$+\left[C_b\gamma N^{\beta-1}+\left(C_L N^{\beta-1}+C_h\right)(1-\gamma)\right]$$
$$*\int_{Q_m}^{\infty}(x-Q_m)h(x;N)dx$$

Subject to Equation 7:

$$C_b\gamma N^{\beta-1}\bar{S}(Q_m)\le K_b \; C_L(1-\gamma)N^{\beta-1}\bar{S}(Q_m)\le K_L \quad (7)$$

where, min $(E(TC(Q_m, N)))$ is the minimum expected annual total cost function.

To find the optimal values Q_m^* *and* N^* which minimize Equation 5 under the two constraints (6), we can use the Lagrange multipliers technique with the Kuhn-Tacker conditions as follows:

$$G(Q_m,N)=\frac{C_r}{N}+\frac{C_o}{N}+C_h\left(Q_m-\mu-\frac{DN}{2}\right)$$
$$+\left[C_b\gamma N^{\beta-1}+\left(C_L N^{\beta-1}+C_h\right)(1-\gamma)\right]$$
$$*\bar{S}(Q_m)+\lambda_b\left(C_b\gamma N^{\beta-1}\bar{S}(Q_m)-K_b\right) \quad (8)$$
$$+\lambda_L\left(C_L(1-\gamma)N^{\beta-1}\bar{S}(Q_m)-K_L\right)$$

where, λ_b and λ_L are the Lagrange multipliers

The optimal values Q_m^* *and* N^* can be calculated by setting the corresponding first partial derivatives of Equation 8 equal to zero, as follows:

$$\frac{\partial G(Q_m,N)}{\partial Q_m}\bigg|_{Q_m=Q_m^*,N=N^*}=0$$

Hence the optimal maximum inventory level is the solution of the following Equation 9:

$$\int_{Q_{mo}}^{\infty}h(x;N)dx=\frac{C_h}{\left((1+\lambda_b)C_b\gamma N^{*\beta-1}+(1-\gamma)\left(1+\lambda_L\right)C_L N^{*\beta-1}+C_h)\right)} \quad (9)$$

Clearly, there is no closed form solution of Equation 9 so we minimize the expected annual total cost numerically. The following algorithm is used:

Step 1: Assume that any initial value for N, put λ_b, λ_L equal to zero then from Equation 9 we have the initial maximum inventory level Q_{m_0} as follows:

$$\int_{Q_{m_0}}^{\infty}h(x;N)dx=\frac{C_h}{C_b\gamma N^{\beta-1}+(1-\gamma)\left(C_L N^{\beta-1}+C_h\right)}$$

Step2: Assume different values for N and varying β then substituting about them in Equation 9 hence we get different values of the maximum inventory level i.e., for N_i we get $Q_{m_i}, i=1,\dots,m$.

Step3: Substituting N_i, Q_{m_i} in Equation 5 then we have different values of total cost until we get the minimum total cost then the correspondence value of N is the optimal value i.e., N^*.

Step4: The procedure is to vary λ_b and λ_L in steps 2 and 3 using N^* until the smallest value of λ_b>0 and λ_L>0 that achieves the constraints for different values of β. Hence we get Q_m^* that gives the minimum annual expected total cost numerically

2.1. Special Cases

Case (1): Let $\beta = 0$, $\lambda_b = 0$ and $\gamma = 1$. Thus Equation 9 becomes:

$$\int_{Q_m}^{\infty}h(x;N)dx=\frac{C_hN}{C_b}$$

This is unconstrained periodic review inventory model for backorder case with constant units of cost, which are the same results as in (Hadley and Whitin, 1963).

Case (2): Let $\beta = 0$, $\lambda_L = 0$ and $\gamma = 0$. Thus, Equation 9 become:

$$\int_{Q_m}^{\infty} h(x;N)dx = \frac{C_h N}{(C_L + C_h n)}$$

This is unconstrained periodic review inventory model for lost sales case with constant units of cost, which are the same results as in (Hadley and Whitin, 1963).

2.2. The Model with Normally Lead Time Demand

In this section, we assume that the lead-time demand follows normal distribution with mean μ and standard deviation σ, i.e.:

$$E(x) = \mu,\ V(x) = \sigma^2\ f(Q_m) = \phi\left(\frac{Q_m - \mu}{\sigma}\right)$$

where, $\phi(z) = N(z; 0,1)$ is the probability density function of the standard normal distribution, $\Phi(z)$ is the cumulative distribution function of the standard normal distribution. Where $z = \frac{Q_m - \mu}{\sigma}$ Substituting in Equation 9, the optimal maximum inventory level is the solution of the following Equation 10:

$$1 - \Phi\left(\frac{Q_m^* - \mu}{\sigma}\right) = \frac{C_h}{\left((1+\lambda_b)C_b\gamma N^{*\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{*\beta-1} + C_h\right)\right)} \quad (10)$$

In addition, the expected number of backorder incurred per period will be:

$$\begin{aligned}\bar{S}(Q_m) &= \int_{Q_m}^{\infty} (x - Q_m) h(x;N^*)dx \\ &= \int_{Q_m}^{\infty} x\,h(x;N^*)dx - Q_m * \left(1 - \int_0^{Q_m} h(x;N^*)dx\right) \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{\frac{Q_m-\mu}{\sigma}}^{\infty} (\sigma z + \mu) e^{-\frac{z^2}{2}} * \sigma dz - Q_m \\ &* \left(1 - \phi\left(\frac{Q_m - \mu}{\sigma}\right)\right) = \sigma\phi\left(\frac{Q_m - \mu}{\sigma}\right) \\ &+ (\mu - Q_m) * \left(1 - \Phi * \left(\frac{Q_m - \mu}{\sigma}\right)\right)\end{aligned} \quad (11)$$

Substituting from Equation 11 to 5 then the expected annual total cost will be:

$$\begin{aligned}\min(E(TC(Q_m, N))) &= \frac{C_r}{N} + \frac{C_o}{N} + C_h\left(Q_m - Dl - \frac{DN}{2}\right) \\ &+ \left[C_b\gamma N^{\beta-1} + (1-\gamma)\left(C_L N^{\beta-1} + C_h\right)\right] * \\ &\left(\sigma\phi\left(\frac{Q_m - \mu}{\sigma}\right) + (\mu - Q_m)\left(1 - \Phi\left(\frac{Q_m - \mu}{\sigma}\right)\right)\right)\end{aligned}$$

Subject to:

$$C_b\gamma N^{*\beta-1}\bar{S}(Q_m) \le K_b\ C_L(1-\gamma)N^{*\beta-1}\bar{S}(Q_m) \le K_L \quad (12)$$

2.3. The Model with Uniform Lead-Time Demand

If the lead-time demand follows the Uniform distribution over range from zero to b then, the probability of the shortage and the expected shortage quantity will be Equation13:

$$\begin{aligned}P(Q_m) &= 1 - F(x) = 1 - \frac{Q_m}{b} Q_m \\ &= b - \frac{C_h b}{\left((1+\lambda_b)C_b\gamma N^{\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{\beta-1} + C_h\right)\right)}\end{aligned} \quad (13)$$

Also, since:

$$\bar{S}(Q_m) = \int_{Q_m}^{\infty} (x - Q_m) f(x)dx = \frac{Qm^2}{2b} + \frac{b}{2} - Q_m \quad (14)$$

Substituting from Equation (13) in (14) by Q_m us get:

$$\bar{S}(Q_m) = \left(\frac{C_h}{\left[(1+\lambda_b)C_b\gamma N^{\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{\beta-1} + C_h\right)\right]}\right)^2 \quad (15)$$

Hence:

$$\begin{aligned}E\left(TC(Q_m, N)\right) &= \frac{C_r}{N} + \frac{C_0}{N} + C_h\left(Q_m - \mu - \frac{DN}{2}\right) \\ &+ \left[C_b\gamma N^{\beta-1} + \left(C_L N^{\beta-1} + C_h\right)(1-\gamma)\right] \\ &* \left(\frac{C_h}{\left[(1=\lambda_b)C_b\gamma N^{\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{\beta-1} + C_h\right)\right]}\right)\end{aligned}$$

Subject to:

$$C_b\gamma N^{\beta-1}\bar{S}(Q_m) \le K_b C_L\ (1-\gamma)N^{\beta-1}\bar{S}(Q_m) \le K_L$$

2.4. The Model with Exponential Lead-Time Demand

If the lead time demand follows the Exponential distribution then:

$$f(x)=\frac{1}{\theta}e^{-\frac{x}{\theta}}, x\geq 0, E(x)=\theta$$

The probability of the shortage and the expected shortage will be Equation 16:

$$P(Q_m)=\int_{Q_m}^{\infty} f(x)dx$$
$$=\frac{C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)} \quad (16)$$

i.e.,

$$P(Q_m)=\int_{Q_m}^{\infty}\frac{1}{\theta}e^{-\frac{x}{\theta}}dx$$
$$=\frac{C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)}$$
$$\Rightarrow \frac{1}{\theta}\int_{Q_m}^{\infty}e^{-\frac{x}{\theta}}dx=e^{-\frac{Q_m}{\theta}}$$
$$=\frac{C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)}$$

By taking In of two sides Equation 17 and 18:

$$\therefore Q_m=-\theta In\left(\frac{C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)}\right) \quad (17)$$

$$\bar{S}(Q_m)=\int_{Q_m}^{\infty}\left(x-Q_m\right)f(x)dx=\theta e^{-\frac{Q_m}{\theta}} \quad (18)$$

Substituting from (17) by Q_m in (18) Equation 19:

$$\bar{S}(Q_m)=\theta e^{-\frac{1}{\theta}Q_m}$$
$$=\theta\left(\frac{C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)}\right) \quad (19)$$
$$=\bar{S}(N)$$

Hence:

$$E\left(TC\left(Q_m,N\right)\right)=\frac{C_r}{N}+\frac{C_o}{N}+C_h\left(Q_m-\mu-\frac{DN}{2}\right)$$
$$+\left[C_b\gamma N^{\beta-1}+\left(C_L N^{\beta-1}+C_h\right)(1-\gamma)\right]$$
$$*\theta\left(\frac{C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)}\right)$$

Subject to:

$$C_b\gamma N^{\beta-1}\bar{S}(Q_m)\leq K_b \quad C_L(1-\gamma)N^{\beta-1}\bar{S}(Q_m)\leq K_L$$

2.5. The Model with Laplace Lead-Time Demand

If the lead time demand follows the Laplace distribution then:

$$f(x)=\frac{1}{2\theta}e^{-\frac{|x-\mu|}{\theta}}, -\infty<x<\infty$$

And $E(x)=\mu$

The probability of the shortage and the expected shortage quantity will be Equation 20:

$$P(Q_m)=$$
$$\int_{Q_m}^{\infty} f(x)dx=\frac{C_h}{\left(\begin{array}{l}\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+\\ (1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\end{array}\right)} \quad (20)$$

But:

$$f(x)=\frac{1}{2\theta}e^{-\frac{|x-\mu|}{\theta}}$$

Then:

$$Q_m=\mu-\theta* In\left(\frac{2C_h}{\left(\left(1+\lambda_b\right)C_b\gamma N^{\beta-1}+(1-\gamma)\left(\left(1+\lambda_L\right)C_L N^{\beta-1}+C_h\right)\right)}\right)$$

Also, since Equation 22:

$$\bar{S}(Q_m)=\int_{Q_m}^{\infty}(x-Q_m)f(x)dx=\frac{1}{2}\theta e^{-\frac{(Q_m-\mu)}{\theta}} \quad (22)$$

Substituting from (21) by Q_m in (22) then.
Since:

$$Q_m = \mu - \theta * In \left(\frac{2C_h}{\left((1+\lambda_b)C_b\gamma N^{\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{\beta-1} + C_h\right)\right)} \right)$$

$$\bar{S}(Q_m) = \frac{1}{2}\theta e^{-\frac{(Q_m-\mu)}{\theta}}$$

Then Equation 23:

$$\bar{S}(Q_m) = \theta \frac{C_h}{\left((1+\lambda_b)C_b\gamma N^{\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{\beta-1} + C_h\right)\right)} \quad (23)$$

Hence:

$$E\left(TC\left(Q_{m,N}\right)\right) = \frac{C_r}{N} + \frac{C_o}{N} + C_h + \left[C_b\gamma N^{\beta-1} + \left(C_L N^{\beta-1} + C_h\right)(1-\gamma)\right] * \theta \frac{C_h}{\left((1+\lambda_b)C_b\gamma N^{\beta-1} + (1-\gamma)\left((1+\lambda_L)C_L N^{\beta-1} + C_h\right)\right)}$$

Subject to:

$$C_b\gamma N^{\beta-1}\bar{S}(Q_m) \le K_b$$
$$C_L(1-\gamma)N^{\beta-1}\bar{S}(Q_m) \le K_L$$

3. STANDARD PARTICLE SWARM OPTIMIZATION (SPSO)

The Particle Swarm Optimization (PSO) algorithm is a population-based search algorithm based on the simulation of the social behavior of birds within the flock and fish school proposed by Kennedy and Eberhart. The population is called a swarm, while the search points are called the particles. The particles are initialized randomly in the search space and have an adaptable velocity; each particle has a memory remembering the best position of the search space it has ever visited. Let we have D-dimensional search space, the swarm is a set of i^{th} particle represented as $X_i = (x_{i1}, x_{i2},..,x_{iD})$ and its velocity for the i^{th} particle is represented as $V_i = (v_{i1}, v_{i2},..,v_{iD})$ The particle swarm optimization concept consists of at each iteration, changing the velocity and location of each particle toward its P_{best} (best value of each particle so far) and g_{best} (best previous position and towards the best particle in the whole swarm) locations according to the following Equation 24 and 25:

$$v_i(t) = wv_{i-1}(t-1) + c_1 r_1 \left(x_{pi} - x_i(t)\right) + c_2 r_2 \left(x_g - x_i(t)\right) \quad (24)$$

$$x_i(t) = cx_i(t-1) + dv_i(t) \quad (25)$$

In the iteration t, the velocity v_i (t) has update to pull the particle i^{th} towards its own best position x_{pi} and the best position for all the particles x_g that has the best fitness value until the preceding generation, r_1, r_2 are random variables uniformly distributed between 0 and 1 this two random values are generated independently, c_1, c_2 are referred to as the cognitive and social parameter and w is the inertia weight. Equation 25 updates each particle's position in the solution hyperspace. Then evaluate the fitness for each particle to find best previous position and global best to update the velocity and the position while the stopping criterion is achieved.

4. FUZZY ADAPTIVE PARTICLE SWARM OPTIMIZATION (FAPSO)

In this section, we introduce a velocity update approach (Liu *et al*., 2007) for the particles in PSO and analyze its effect on the particle's behavior during the D-dimensional search space. One of the main effects is the premature convergence that occurs when the v_i arrived to zero or near to zero, but this does not mean that the particle arrives to the global or local best particle but mean the best position particle. In the FAPSO (Elhefnawy *et al*., 2007), the minimum velocity v_{c1} can be tuned adaptively by using the fuzzy control parameter α in the solution procedure to overcome the previous case. If a particle's velocity decreases to a threshold v_{c1}, a new velocity is assigned using Equation 26 to drive those lazy particles and let them explorebetter solutions. Thus, we present the FAPSO using the following velocity update:

$$v = \begin{cases} v_i & if \left|v_i\right| \geq v_{cl} \\ u(-1,1)v_{\max}/\rho & if \left|v_i\right| < v_{cl} \end{cases} \quad (26)$$

where, $u(-1, 1)$ is the random number, uniformly distributed with the interval [-1, 1] and ρ is the scaling factor to control the domain of the particle's oscillation according to v_{max} where The value of v_{max} is $\rho \times s$, with $0.1 \leq \rho \leq 1.0$ and is usually chosen to be s, i.e., $\rho = 1$. **Figure 2** illustrates the trajectory of a single particle in FAPSO using the fuzzy control parameter α. Also, shows the effects of the different fuzzy control parameter α on the behavior of the solution procedure, respectively.

The Procedure of the FAPSO can be Explained as Follows:

Step1: Generate a set of initial solutions of the Probabilistic $<Q_m, N>$ Inventory Model with Varying Mixture Shortage

Step2: Constructing the membership function for particle's velocity

Step3: Determine the control parameter α to obtain v_{cl} that may cause the premature convergence

Step4: Evaluate the fitness function of each particle

Step5: If the particle does not remain in feasible solution region (divergence particle), discard it and mutated again with $x_i = x_{pi}$ go to step 8

Step6: The particle's velocity can be updated based on the following equation:

$$v = \begin{Bmatrix} v_i(t) & if \left|v_i(t)\right| \geq v_{c1} \\ u(-1,1)v_{\max}/\rho & if \left|v_i\right| \prec v_{c1} \end{Bmatrix}$$

Where:

$$v_i(t) = wv_i(t-1) + c_1 r_1 \left(x_{pi} - x_i(t)\right) + c_2 r_2 \left(x_g - x_i(t)\right)$$

Step7: The position of each particles can be updated according to the following equations $x_i(t) = cx_i(t-1) + dv_i(t)$

Step8: Save the new fitness values in the repository

Step9: If the no. of generation reached go to step 10. Otherwise, go to step 4.

Step10: Stop.

Figure 3 represents the flow chart for the suggested multi-objective FAPSO algorithm**.**

5. NUMERICAL EXAMPLE

Consider the following data for solving the probabilistic periodic review $<Q_m, N>$ inventory model with mixture shortage given in **Table 1**.

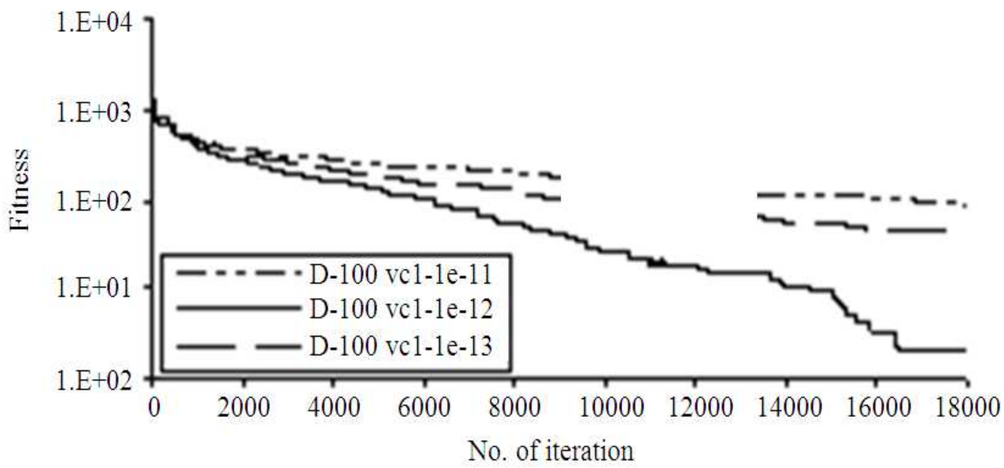

Fig. 2. The behavior of the algorithms at different α

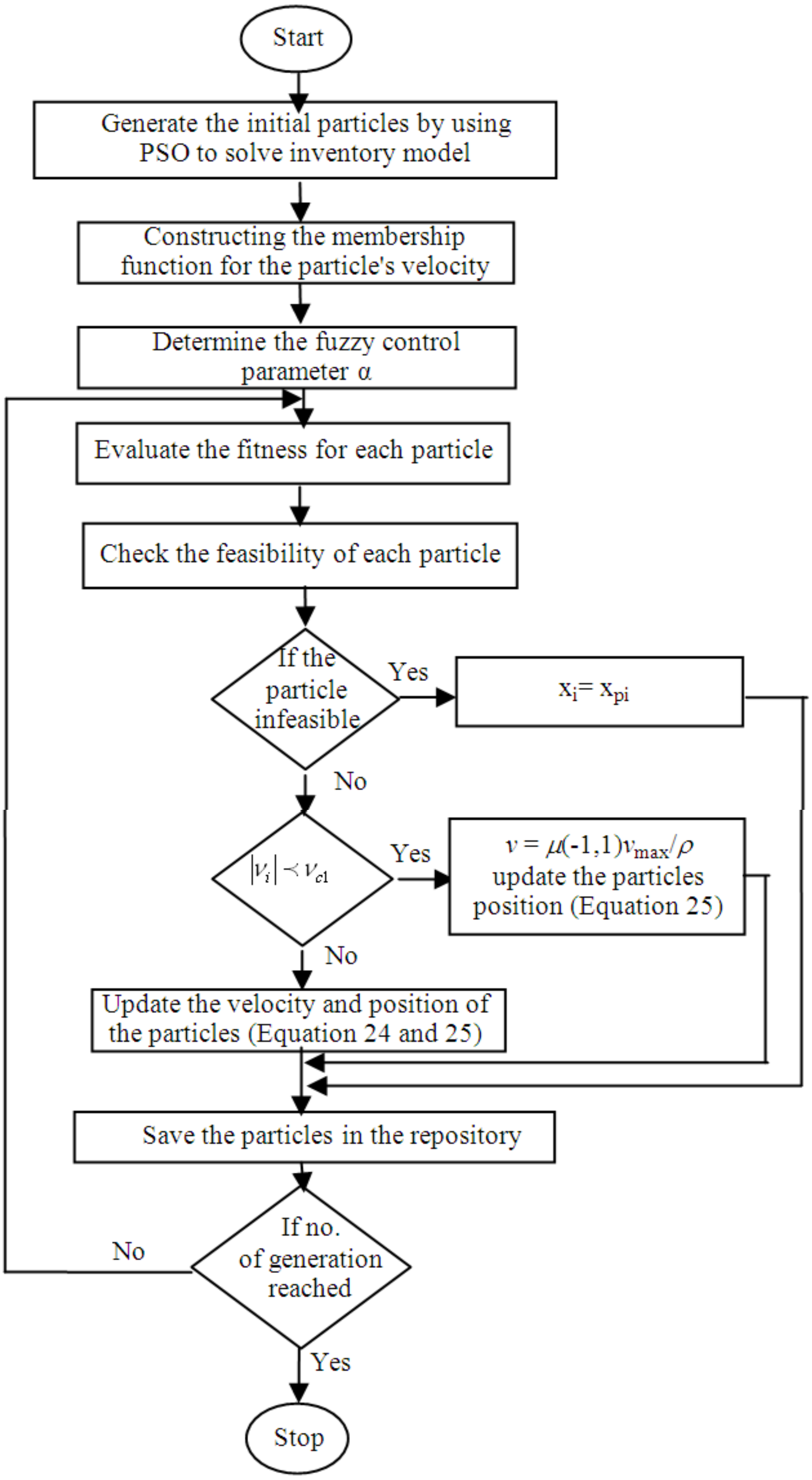

Fig. 3. Flow chart of the suggested FAPSO algorithm

A warehouse follows a policy of reviewing all items periodically every 1.94 month (N^*). The lead-time L is nearly constant and its value is 6 months. A fraction of unsatisfied demand that will be backorder γ is 0.56.

Each simulation run was carried by using the following parameters when solved by FAPSO approach:

Number of generations	=	500
Population size	=	80
Self-recognition coefficient c_1	=	1.49
Social coefficient c_2	=	1.49
Inertia weight ω	=	0.9

Each optimization experiment was run 10 times with different random seeds. The optimal review period (N^*), optimal maximum inventory level (Q_m^*) and constant real numbers (β) which will minimize the expected annual total cost E (TC)\ recorded in the **Table 3-9**.

Consider the average demand rate has been constant over time given by the value of 600 units per year for the normal distribution and 100 units per year for the remaining distribution. It is desired to determine to optimal values Q_m^*, N^* and the minimum total cost in the following cases:

When lead time demand follows normal distribution under the following constrained Equation 27:

$$\left.\begin{array}{l} E(BC) \le 15 \\ E(LC) \le 11.785 \end{array}\right\} \tag{27}$$

When lead time demand follows uniform distribution for $0 \le x \le 50$ under the following constrained Equation 28:

$$\left.\begin{array}{l} E(BC) \le 0.96 \\ E(LC) \le 0.747 \end{array}\right\} \tag{28}$$

When lead time demand follows exponential distribution for $x \ge 0, \theta = 25$ under the following constrained Equation 29:

$$\left.\begin{array}{l} E(BC) \le 40 \\ E(LC) \le 31.429 \end{array}\right\} \tag{29}$$

When lead time demand follows Laplace distribution for $\mu = 25$, $\theta = 10.206$ under the following constrained Equation 30:

$$\left.\begin{array}{l} E(BC) \le 16 \\ E(LC) \le 12.672 \end{array}\right\} \tag{30}$$

5.1. Test of the Problem Using Lagrange Multiplier Technique and FAPSO

To make sure (examine) the efficiency and fitness of the algorithm of Fuzzy Adaptive Particle Swarm Optimization (FAPSO) we aim at fixing N, β between the interval [0,1] and test the results between FAPSO and Lagrange multiplier technique. The FAPSO give us results close to the results of Lagrange multiplier and give us an impression for the validity of the algorithm but in this case we restrict the FAPSO algorithm. For this reason, we examine the FAPSO algorithm when N, β are variables and compare the results of FAPSO with the similar of Lagrange multiplier technique using Mathematica program. Hence we found that FAPSO gives results better than Lagrange multiplier technique, which discussed in section 5.2.

Table 2-5 represent the results of two techniques at N = 1.94 month and a constant real number selected to provide the best fit of Estimated expected cost function (β) between 0.1 and 0.9.

5.2. Comparative study

5.2.1. The Solution for the (Normal Distribution)

Let the demand in the time $L+N$ can be represented quite well by a normal distribution with mean $D(L+N) = 600(L+N)$ and variance $\sigma^2(L+N) = 900(L+N)$.

It is desired to determine to optimal values Q_m^*, N^* and the minimum total cost:

- The expected demand in time $L+N$ is $E(x) = 397.002$
- The variance of the demand in this time $\sigma^2(L+N) = 595.5$
- Then the standard deviation is $\sigma\sqrt{(L+N)} = \sqrt{595.5} = 24.4029$
- Hence the results using Mathematica program can be summarized as follows

The optimum values for different values of β and the total cost based on the Lagrange multiplier technique under two constraints when the lead time demand follows normal distribution with mean μ and standard deviation σ is given by **Table 6**.

However, the results using Fuzzy Adaptive Particle Swarm Optimization can be summarized as follows.

Consider that N, β *and* $Q_m^{\backslash}$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach under two constraints when the lead-time demand follows normal distribution with mean μ and standard deviation σ is given by **Table 7**. Moreover, clarify of the results is shown in **Fig. 4**.

5.3. The Solution for the (Uniform Distribution)

$$Let\ f(x)=\frac{1}{50}\ hence\ E(x)=25$$

Also:

$$\bar{S}(Q_m)=\frac{Q_m^2}{2b}+\frac{b}{2}-Q_m=\frac{Q_m^2}{100}+\frac{50}{2}-Q_m$$

The optimum values of the maximum inventory level and the total cost based on the Lagrange multiplier technique when the lead-time demand follows uniform distribution is given by **Table 8**.

Consider that $N,\beta\ and\ Q_m^{\backslash}$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach when the lead-time demand follows uniform distribution is given by **Table 9**. Moreover, clarify of the results displayed in **Fig. 5**.

5.4. The Solution for the (Exponential Distribution)

Let:

$$f(x)=\frac{1}{25}e^{-\frac{x}{25}},x\geq 0,\theta=25$$

Hence:

$$E(x)=25\ \bar{S}(Q_m)=\theta e^{-\frac{Q_m}{\theta}}=25e^{-\frac{Q_m}{25}}$$

The optimum values of the maximum inventory level and the total cost based on the Lagrange multiplier technique when the lead-time demand follows exponential distribution is given by **Table 10**.

Consider that $N,\beta\,and\,Q_m^{\backslash}$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach when the lead time demand follows Exponential distribution is given by **Table 11** and explain of the results is exhibit in **Fig. 6**.

5.5. The Solution for the (Laplace Distribution)

$$E(x)=25\ \bar{S}(Q_m)=\frac{1}{2}\theta e^{\frac{-(Q_m-\mu)}{\theta}}=\frac{1}{2}*10.206*e^{\frac{-(Q_m-25)}{10.206}}$$

The optimum values of the maximum inventory level and the total cost based on the Lagrange multiplier technique when the lead-time demand follows Laplace distribution is given by **Table 12**.

Consider that $N,\beta\ and\ Q_m^{\backslash}$ are variables then the optimum values of the time between reviews, the maximum inventory level and the total cost using FAPSO approach when the lead-time demand follows Laplace distribution is given by **Table 13**. Moreover, illustrate of the results is shown in **Fig. 7**.

5.6. Remark

From the previous results it becomes clear that when we Use FAPSO approaches it leads toward the global optimum instead of trapping into local peaks in all the pervious distributions.

Table 1. Presents the value of the parameters

Parameters	Value
C_o+C_r	25 $
C_h	3 $
C_b	25 $
C_L	25$

Table 2. The results using mathematica program and FAPSO for Normal distribution

β	Q_m^*	E(TC)	$Q_m^{*\vert}$	E(TC)|
0.1	445.715	473.31 $	407.9091	202.3958
0.2	444.109	468.567	408.2650	208.3604
0.3	442.455	463.655	408.2995	223.4091
0.4	440.670	458.393	408.3293	240.7190
0.5	438.864	453.068	408.4040	253.1674
0.6	437.001	447.598	408.4786	264.2640
0.7	435.105	442.051	408.5786	273.3417
0.8	433.194	436.481	408.0180	302.4184
0.9	431.172	430.612	407.9785	295.6521

Table 3. The results using Mathematica program and FAPSO for Uniform distribution

β	Q_m^*	E(TC)	$Q_m^{*\backslash}$	E(TC)|
0.1	48.85270	203.657$	49.29800	205.7859
0.2	48.74330	203.333	48.71286	205.0256
0.3	48.62340	202.977	49.46144	205.8740
0.4	48.49210	202.588	48.62533	204.5115
0.5	48.34823	202.163	48.87756	204.6693
0.6	48.19070	201.697	48.24354	203.5291
0.7	48.01810	201.188	48.24241	203.3445
0.8	47.82910	200.631	47.97995	202.6509
0.9	47.62200	200.022	48.01763	202.4603

Table 4. The results using Mathematica program and FAPSO for Exponential distribution

β	Q_m^*	E(TC)	$Q_m^{*\backslash}$	E(TC)\
0.1	95.2259	413.223 $	94.93518	411.9810
0.2	90.6704	399.703	89.87002	400.7742
0.3	86.1149	386.212	88.49079	387.0650
0.4	81.5594	372.756	82.73436	373.4911
0.5	77.0039	359.342	83.55936	362.7138
0.6	72.4485	345.979	76.05411	345.3453
0.7	67.8929	332.676	73.58664	335.8071
0.8	63.3375	319.446	65.25027	321.6515
0.9	58.7820	306.303	57.99037	304.7468

Table 5. The results using Mathematica program and FAPSO for Laplace distribution

β	Q_m^*	E(TC)	$Q_m^{*\backslash}$	E(TC)\
0.1	57.009	255.276$	64.64356	251.1807
0.2	55.1493	249.755	62.81299	245.6890
0.3	53.2895	244.246	58.65988	233.2296
0.4	51.4297	238.751	58.92988	234.0396
0.5	49.5700	233.273	55.32898	223.2369
0.6	47.7103	227.815	51.73142	212.4443
0.7	45.8504	222.381	50.58961	209.0188
0.8	43.9908	216.977	50.31293	208.1888
0.9	42.1310	211.607	50.13726	207.6618

Table 6. The optimal results of model 2-1

β	λ_b^*	λ_L^*	Q_m^*	E(TC)
0.1	0.002	0.00260	445.715	473.31 $
0.2	0.035	0.02547	444.109	468.567
0.3	0.075	0.03815	442.455	463.655
0.4	0.107	0.03818	440.670	458.393
0.5	0.145	0.03820	438.864	453.068
0.6	0.182	0.03820	437.001	447.598
0.7	0.224	0.03820	435.105	442.051
0.8	0.276	0.03815	433.194	436.481
0.9	0.3147	0.03820	431.172	430.612

Table 7. The results using FAPSO

β	N^*	$Q_m^{*\backslash}$	E (TC)\
1.35E-02	0.27994	483.6817	91.04408
0.254816	0.264483	475.0849	110.66580
0.121196	0.276644	481.7898	117.30190
3.50E-03	0.279011	482.6383	118.09550
2.69E-03	0.259179	469.9347	134.35520
5.88E-03	0.282823	484.7993	134.78630
0.289517	0.180891	422.2667	139.28720
0.338156	0.296493	494.7425	166.28250
4.63E-02	0.281428	481.3388	315.68550

Table 8. The optimal results of the model 2-2

β	λ_b	λ_L	$Q_m^{*\backslash}$	E(TC)
0.1	0.00450	0.00382	48.85270	203.657 $
0.2	0.17380	0.00382	48.74330	203.333
0.3	0.35870	0.00382	48.62340	202.977
0.4	0.56080	0.00382	48.49210	202.588
0.5	0.78155	0.00376	48.34823	202.163
0.6	1.02270	0.00382	48.19070	201.697
0.7	1.28620	0.00382	48.01810	201.188
0.8	1.57390	0.00382	47.82910	200.631
0.9	1.88780	0.00382	47.62200	200.022

Table 9. The results using FAPSO

N	β	$Q_m^{*\backslash}$	E(TC)\
0.855797	0.630314	58.38030	20.50866
0.693346	0.197418	54.16220	25.58115
0.578352	0.481714	42.89127	27.59362
0.560751	0.649507	47.76734	30.36465
0.664484	0.147211	53.94762	30.51989
0.59093	0.303433	51.05595	32.25129
0.588301	0.120997	52.01549	36.96916
0.541767	0.097877	50.53540	41.61468
0.549307	0.124951	51.50322	43.60950
0.758902	0.133017	32.24886	45.07503

Table 10. The optimal results of the model 2-3

β	λ_b^*	λ_L^*	$Q_m^{*\backslash}$	E(TC)
0.1	0.041000	0.038190	95.2259	413.223 $
0.2	0.037350	0.038180	90.6704	399.703
0.3	0.032960	0.038180	86.1149	386.212
0.4	0.027700	0.038180	81.5594	372.756
0.5	0.021380	0.038180	77.0039	359.342
0.6	0.013810	0.038180	72.4485	345.979
0.7	0.004710	0.038180	67.8929	332.676
0.8	0.003805	0.025460	63.3375	319.446
0.9	0.000720	0.012728	58.7820	306.303

Table 11. The results using FAPSO

β	N^*	$Q_m^{*\backslash}$	E(TC)\
0.4058053	0.97973	57.43209	42.78574
0.935998	0.98157	59.75097	42.85972
0.624991	0.95466	49.19647	49.08882
9.81E-02	0.95122	54.91412	49.72275
0.642026	0.99140	69.49759	50.95656
0.471227	0.93516	54.96149	51.88449
0.229297	0.95286	65.71236	54.66653
0.31015	0.92877	49.52028	56.46565
0.203667	0.59067	76.79328	154.6951

Table 12. The optimal results of the model 2-4

β	λ_b^*	λ_L^*	Q_m^*	E(TC)
0.1	0.106650	0.038200	57.0090	255.276$
0.2	0.103000	0.003818	55.1493	249.755
0.3	0.098600	0.003818	53.2895	244.246
0.4	0.093330	0.003818	51.4297	238.751
0.5	0.087020	0.003818	49.5700	233.273
0.6	0.079460	0.003818	47.7103	227.815
0.7	0.070340	0.003818	45.8504	222.381
0.8	0.059436	0.003818	43.9908	216.977
0.9	0.04635	0.003818	42.1310	211.607

Table 13. The results using FAPSO

β	N^*	$Q_m^{*\backslash}$	E(TC)\
0.235573	0.471201	34.38432	10.53058
0.190189	0.901405	64.60248	11.33112
0.538946	0.848705	61.44213	11.47725
0.629335	0.593022	44.74105	12.42682
0.680409	0.400022	30.52506	19.13798
0.350657	0.616576	48.75258	19.31785
0.315997	0.462833	37.51333	22.13034
0.61585	0.594566	49.71379	27.00388
2.31E-02	0.841329	93.55012	109.16580
0.895867	0.668046	95.88036	149.85670

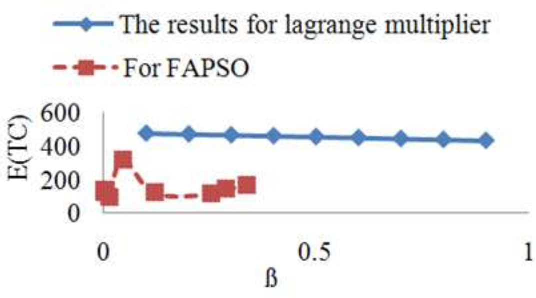

Fig. 4. Display the results of normal distribution

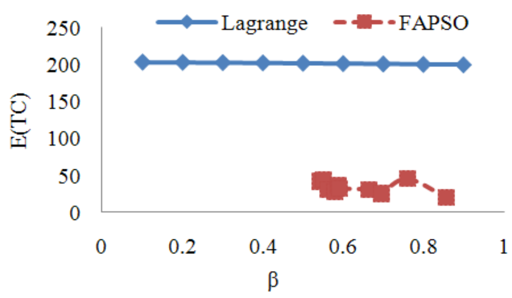

Fig. 5. Display the results of uniform distribution

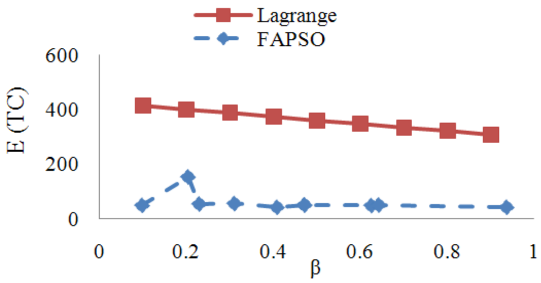

Fig. 6. Exhibit the results of exponential distribution

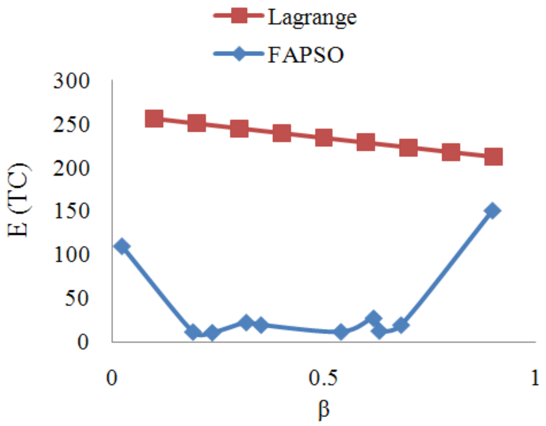

Fig. 7. Illustrate the results of Laplace distribution

6. CONCLUSION

In this study, we developed a probabilistic Single-Item Single-Source (SISS) inventory model with varying mixture of backorders and lost sales under two restrictions, which the first on the expected backorder cost and the other on the expected lost sales cost. We reached the optimal review period and optimal maximum inventory level that minimized the expected annual total cost under constraints using Lagrange multiplier technique and the Fuzzy Adaptive Particle Swarm Optimization (FAPSO). We overcome the problems that meet us when we use Lagrange multiplier technique by using FAPSO whereas most algorithms tend to be stuck to a sub-optimal solution, an algorithm efficient in solving one optimization problem may not be efficient in solving another one and these techniques such as Lagrange multiplier technique are useful over a relatively narrow range. FAPSO proved good results and performance when applied to solve complexities problems. Using FAPSO algorithm in the inventory model promise to achieve the global solution with decreasing the number of iteration that required for arrive to it. The algorithm has been tested using a numerical example; the results show the algorithms described in this study perform well.

In the future, we want to increase the number of constraints to contain all the costs and implement Fuzzy Adaptive Particle Swarm Optimization (FAPSO) on the constrained <$Q_{m,}N$> modelwith mixture varying shortage when all components of costs are fuzzy number

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