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Research Article

RK4 Differential Equation to improve QoS in an eUTRAN LTE Network: A Deterministic vs. Stochastic Approach

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Abstract: Improving Quality of Service (QoS) in an eUTRAN LTE network is essential to guarantee optimal performance in the face of dynamic variations in user traffic and radio channel interference. In this context, this paper explores the use of 4^{th}-order Runge-Kutta differential equations (RK4) to model and optimize radio resource management, by comparing a deterministic approach, which assumes stable and predictable traffic, with a stochastic approach, which incorporates random network fluctuations. The central problem lies in the difficulty of anticipating and effectively managing network congestion, affecting resource allocation, bandwidth and transmission delay. Through a 24 h simulation, the results show that the deterministic approach predicts a resource allocation of up to 90% at peak times, a bandwidth occupancy of 80 Mbps and a maximum latency of 120 ms, offering stable but limited management in the face of unforeseen events. In contrast, the stochastic approach reveals fluctuations of ±5% in resource allocation, variations of ±10 Mbps in bandwidth and a transmission delay of up to 130-140 ms, reflecting a better representation of real network conditions. These results demonstrate that the deterministic approach offers a predictable view of the network, while the stochastic approach enables better dynamic adaptation, essential for anticipating congestion and adjusting resources in real time.

Keywords: 4G/LTE, Quality of Service (QoS), RK4 Differential Equations, Deterministic vs. Stochastic Approach, LTE Network Congestion, Predictive Optimization

Introduction

The evolution of mobile networks towards Long Term Evolution (LTE) technology has led to significant improvements in Quality of Service (QoS), offering higher data rates, reduced latency, and better management of radio resources. However, optimizing QoS in an evolved Universal Terrestrial Radio Access Network (eUTRAN) network remains a major challenge due to dynamic variations in user traffic and interference inherent in the radio channel. The use of differential equations to model the evolution of network resources and transmission performance proves to be a relevant approach. This study proposes a model based on Runge-Kutta 4^{th}-order numerical integration (RK4) to analyze and optimize QoS in an LTE network. Two complementary approaches are explored: A deterministic approach, where variations in traffic and interference are assumed to be constant and predictable, and a stochastic approach, where these fluctuations are modeled by random processes reflecting real network dynamics. Through numerical simulations, we compare these two methodologies to highlight their respective impacts on radio resource allocation, bandwidth occupation, and transmission delay. The results show that while the deterministic approach provides a stable and predictable view of the network, it does not consider the vagaries of user behavior and channel conditions. On the other hand, the stochastic approach, although more complex to model, offers a better representation of real-life scenarios and enables network congestion to be anticipated. The aim is to propose a dynamic radio resource optimization strategy that not only improves QoS but also predicts periods of congestion in order to proactively adjust resource allocation. This work thus paves the way for more efficient and adaptive management of LTE networks, by integrating advanced mathematical models coupled with simulation techniques for greater resilience in the face of traffic fluctuations. In the following, a review of the literature is presented, with a discussion of the QoS issue and the evolution of mobile radio standards and their impact on QoS. In the following, the RK4 differential system applied to an LTE network will be developed, whose state variables are the QoS characteristics to be improved.

Literature Review

The notion of QoS in a mobile network has always been a problem that planners should solve before deploying a mobile network. With this in mind, researchers' work on cell planning in mobile networks focused on cell coverage according to mobile technology engineering and to solve this problem two approaches were proposed for the Second Generation of Mobile networks (2G-GSM): The first approach, called Coverage Based Design, consists of minimizing the number of Base Stations (BTS) to be deployed and finding their position so that the Signal-to-Interference Ratio (SIR) received at the mobile terminal is high enough to satisfy demand (Sherali and Pendyala, 1996). The second approach, called Demand Based Design, transforms the minimization problem of the first approach into a problem of maximizing the assumed fixed number of base stations (Tutschku, 2002).

In this context, Buddendick et al. (2005) developed optimization algorithms and models for joint uplink/downlink planning, controlling the strength of the Signal-to-Interference Ratio (SIR) of the radio subsystem of a UMTS network. An evolution of UMTS technology with Internet Protocol (IP) and its Multi-Protocol Label Switching (MPLS) mechanisms in 3G mobile networks enabled Pasandideh & Hilaire (2014) to design a model to solve the problem of planning an all-IP UMTS network according to realistic traffic by developing a local search heuristic. With the advent of Long-Term Evolution (4G-LTE) technology, mobile networks are using OFDMA coding schemes in the downlink and SC-FDMA in the uplink direction driven by HARQ-type error recovery algorithms and turbo codes (Holma and Toskala, 2012). This LTE technology allows new parameters to be considered in the sizing of base stations in mobile networks; with this in mind, aiming to satisfy cell coverage Hakim et al. (2016) determine a cell optimization model based on the spatio-temporal variation of users and traffic density at specific times of the day. Other works go even further, proposing the determination of optimal base station positions through the appropriate sizing of Tracking Areas (TAs), with the aim of minimizing the cost of signaling during user mobility (Safa and Ahmad, 2015). On the other hand, some authors present a method of dynamically controlling the power of eNodeBs to optimize coverage in LTE networks (Chen et al. 2019), which is not the case for Xu et al. who focus on optimizing capacity and coverage in LTE-A networks through eNodeB sectorization and power control techniques (Xu et al., 2012). Jaloun et al., in 2011, proposed an integer optimization based on genetic programming developed by evaluating the signal-to-interference plus noise ratio (Jaloun et al., 2011). A year later, in 2012, the work of Lee et al. advocates instead eNodeB scheduling with interference coordination in LTE networks (Lee et al., 2012), Finally, Djomadji et al. 2023 designed an effective machine learning model that takes certain Key Performance Indicators (KPIs) as input, such as traffic data, RRC, simultaneous users, etc., for each eNodeB per hour and per day and accurately predicts the number of RRC resources required, traffic losses and financial losses for the mobile network operator (Djomadji et al., 2023).

This work highlights the importance of QoS in mobile networks to satisfy user needs and this study explores the use of Runge-Kutta fourth-order differential equations to improve the QoS of an LTE network, with the aim of providing a mathematical answer to this problem.

Mathematical Model

Quality of Service (QoS) enhancement in an Evolved Universal Terrestrial Radio Access Network (E-UTRAN) based on LTE technology can be modeled using Runge-Kutta 4^{th}-order differential equations (RK4). This approach can be used to approximate the dynamics of data flows, queue management, and radio resource allocation.

A differential equation in the network is of the following form:

$$\frac{dX}{dt} = f(X, t) \tag{1}$$

Where:

- X: represents LTE network status variables such as radio resource allocation, bandwidth occupation, transmission delay, etc
- $f(X, t)$: is a function describing the evolution of resources in the network
- Thus, considering radio resource allocation, bandwidth occupation, and transmission delay as the QoS variables to be improved, we have:
- $X_1(t)$: Radio resource allocation (%)
- $X_2(t)$: Bandwidth utilization (Mbps)
- $X_3(t)$: Transmission delay (ms)

The system of differential equations representing the evolution of quality of service (QoS) in an LTE network as a function of time is:

$$\frac{dX_1}{dt} = \lambda_r - \mu_r X_1 \tag{2}$$

$$\frac{dX_2}{dt} = \gamma X_1 - \delta X_2 \tag{3}$$

$$\frac{dX_3}{dt} = \alpha X_2 - \beta X_3 \tag{4}$$

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Where:

- λ_r: Radio resource allocation rate (linked to user demand)
- μ_r: Radio resource release factor (linked to output rate)
- γ: Conversion of radio allocation to bandwidth occupancy
- δ: Bandwidth dissipation factor (related to network traffic)
- α: Influence of bandwidth occupancy on delay
- β: Transmission delay control

In an RK4 simulation, we use the following iteration to estimate(X,t):

$$X_{n+1} = X_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \tag{5}$$

With:

$$k_1 = hf\left(X_{n,}t_n\right) \tag{6}$$

$$k_2 = hf\left(X_{n,} + \frac{k_1}{2}, t_n + \frac{h}{2}\right) \tag{7}$$

$$k_3 = hf\left(X_{n,} + \frac{k_2}{2}, t_n + \frac{h}{2}\right) \tag{8}$$

$$k_4 = hf = f\left(X_{n,} + k_3, t_n + h\right) \tag{9}$$

Where:

- h: Simulation time step (accuracy of approximation)
- k_1: Initial slope of the differential equation
- k_2: Intermediate correction after half a time step
- k_3: Second intermediate correction based on k_2
- k_4: Final slope after one full step

X is updated by combining these values with specific weights to ensure accurate estimation in the LTE network.

Application of RK4 in an LTE Network

In an eUTRAN network, by applying RK4 to the differential Eqs. in (2-4), we have the following differential equations to model the different QoS variables:

For $X_1(t) = \frac{dX_1}{dt}$ (Radio resource allocation):

- $X_1^{(n+1)}(t) = X_1^{(n)}(t) + \frac{1}{6}\left(k_1^{(1)} + 2k_2^{(1)} + 2k_3^{(1)} + k_4^{(1)}\right)$ (10)
- $k_1^{(1)} = h\left(\lambda_r - \mu_r X_1^{(n)}(t)\right)$ (11)
- $k_2^{(1)} = h\left(\lambda_r - \mu_r\left(X_1^{(n)}(t) + \frac{k_1^{(1)}}{2}\right)\right)$ (12)
- $k_3^{(1)} = h\left(\lambda_r - \mu_r\left(X_1^{(n)}(t) + \frac{k_2^{(1)}}{2}\right)\right)$ (13)
- $k_4^{(1)} = h\left(\lambda_r - \mu_r\left(X_1^{(n)}(t) + k_3^{(1)}\right)\right)$ (14)

For $X_2(t) = \frac{dX_2}{dt}$ (Bandwidth utilization):

- $X_2^{(n+1)}(t) = X_2^{(n)}(t) + \frac{1}{6}\left(k_1^{(2)} + 2k_2^{(2)} + 2k_3^{(2)} + k_4^{(2)}\right)$ (15)
- $k_1^{(2)} = h\gamma\left(X_1^{(n)}(t) - \delta X_2^{(n)}(t)\right)$ (16)
- $k_2^{(2)} = h\left(\gamma\left(X_1^{(n)}(t) + \frac{k_1^{(1)}}{2}\right) - \delta\left(X_2^{(n)}(t) + \frac{k_1^{(2)}}{2}\right)\right)$ (17)
- $k_3^{(2)} = h\left(\gamma\left(X_1^{(n)}(t) + \frac{k_2^{(1)}}{2}\right) - \delta\left(X_2^{(n)}(t) + \frac{k_2^{(2)}}{2}\right)\right)$ (18)
- $k_4^{(2)} = h\left(\gamma\left(X_1^{(n)}(t) + k_3^{(1)}\right) - \delta\left(X_2^{(n)}(t) + k_3^{(2)}\right)\right)$ (19)

For $X_3(t) = \frac{dX_3}{dt}$ (transmission delay variation):

- $X_3^{(n+1)}(t) = X_3^{(n)}(t) + \frac{1}{6}\left(k_1^{(3)} + 2k_2^{(3)} + 2k_3^{(3)} + k_4^{(3)}\right)$ (20)
- $k_1^{(3)} = h\left(\alpha X_2^{(n)}(t) - \beta X_3^{(n)}(t)\right)$ (21)
- $k_2^{(3)} = h\left(\alpha\left(X_2^{(n)} + \frac{k_1^{(2)}}{2}\right) - \beta\left(X_3^{(n)}(t) + \frac{k_1^{(3)}}{2}\right)\right)$ (22)
- $k_3^{(3)} = h\left(\alpha\left(X_2^{(n)} + \frac{k_2^{(2)}}{2}\right) - \beta X_3^{(n)}(t) + \frac{k_2^{(3)}}{2}\right)$ (23)
- $k_4^{(3)} = h\left(\alpha\left(X_2^{(n)}(t) + k_3^{(2)}\right) - \beta X_3^{(n)}(t) + k_3^{(3)}\right)$ (24)

This differential equation model (10-24) enables:

- Analyze the dynamics of radio resources in an LTE network over time (10-14)
- Simulate bandwidth and delay variations under different network loads over time (15-19)
- Optimize resource management to improve Quality of Service (QoS) over time (20-24)

Simulation Hardware and Software

- Laptop (i7, 16GB RAM, 1TB SSD) running Linux (Ubuntu for srsRAN/OpenAirInterface)
- Rooted Android 4G smartphone + GNetTrack
- Netgear 4G LTE Modem (LB2120)
- USB GPS
- Python (Jupyter + scikit-learn + matplotlib)
- MATLAB R2023a /Simulink

Data Selection

Real LTE data collected via a combination of network instruments, mobile devices, field tools, and analytical tools over one day for resource allocation (%), bandwidth (Mbps), and transmission delay (ms)

Initial Conditions and Input Parameters for LTE Model Simulation

Condition Initial (t = 0)

The initial values of the state variables ($X_1^{(n)}(t), X_2^{(n)}(t), X_3^{(n)}(t)$ must represent an average or realistic state of the LTE network at start-up :

Total number of eNodeB N = 10NeNB = 10 (An urban LTE network with several base stations)

- $X_1^{(n)}(t=0) = 20\%$: Initial allocation of radio resources (%)
- $X_2^{(n)}(t=0) = 15Mbps$: Initial bandwidth utilization (Mbps)
- $X_3^{(n)}(t=0) = 40ms$: Initial transmission delay (ms)

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Mathematical Model Input Parameters (t = 0)

- Radio resource allocation parameters
  - Radio resource allocation rate (%/s): $\lambda_r = 8 + \xi$, ξ: Stochastic variable representing the variability of user traffic and follows a normal $\mathrm{N}(0, 2)$ distribution for fluctuations of 2% around the mean
  - Radio resource release factor (%/s): $\mu_r = 0.3$ (deterministic fixed factor)
- Bandwidth occupancy parameters
  - Conversion of radio allocation to bandwidth occupancy (Mbps/%): $\gamma = 0.7$(deterministic fixed factor)
  - Bandwidth dissipation (Mbps/s): $\delta = 0.4 + \eta$ où η: Stochastic variable simulating interference and network losses following a uniform distribution between [-0.1, 0.1] in (Mbps/s)
- Transmission delay parameters
  - Influence of bandwidth on delay (ms/Mbps): $\alpha = 0.2$deterministic fixed factor)
  - Regulation of transmission delay (ms/s): $\beta = 0.1 + \zeta$where ζ is the stochastic Variable simulating dynamic network congestion following a normal distribution N (0,0.02) for slight variations

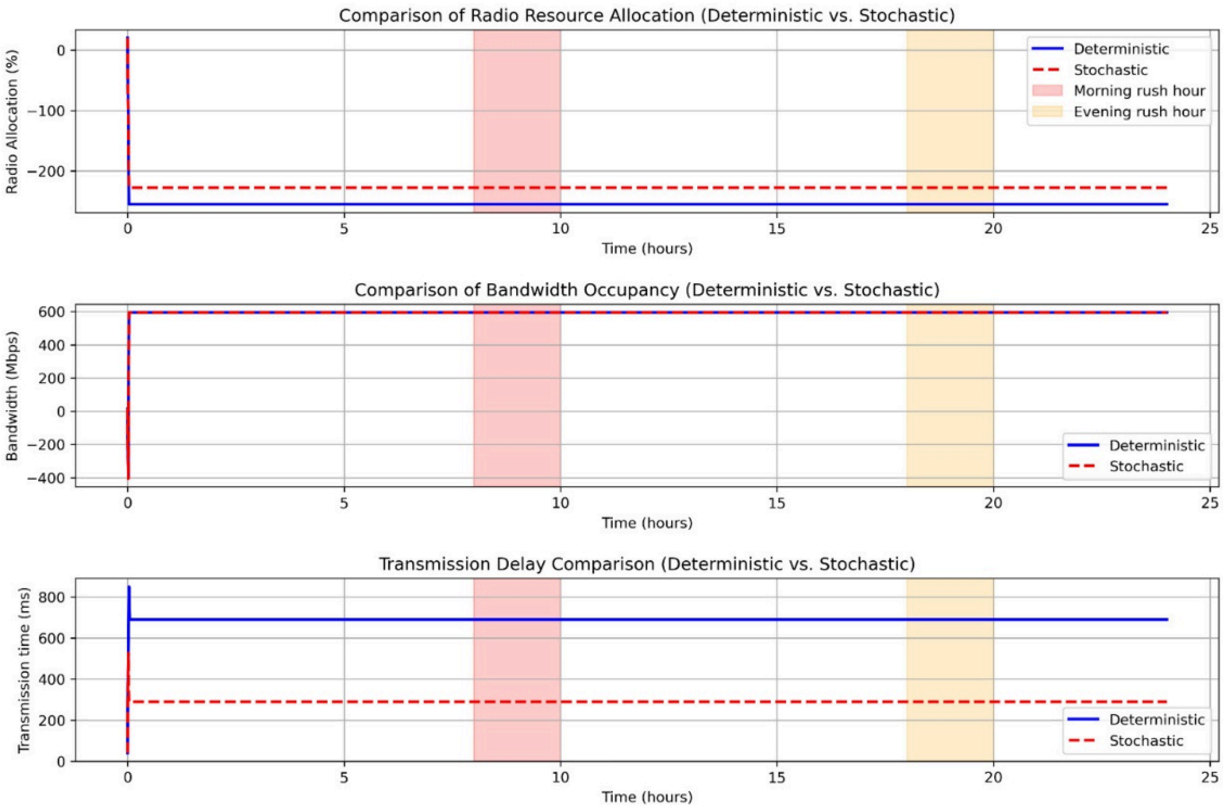

Fig. 1: Evolution of key LTE network variables as a function of time under two scenarios: deterministic and stochastic

Simulation Parameters

- Initial time:$t = 0s$
- Final time: $t_f = 84600$ s (durée de simulation pour observer la stabilisation du réseau)
- Time step: $h = 60$s (accuracy of RK4 method)
- Stochastic simulation: Includes ξ, η, ζ to model uncertainty
- Deterministic simulation: Uses only fixed mean values with no fluctuation
- Deterministic simulation provides a stable model that describes the dynamics of the LTE network without disturbances
- Stochastic simulation allows analysis of real variations and the impact of randomness on quality of service (QoS).

Results

The three graphs in Figure 1 show the evolution of key LTE network variables as a function of time under two scenarios: Deterministic and Stochastic, for a 24-hour LTE network. We analyze three key metrics:

- $X_1^{(n)}$: Radio resource allocation (%)
- $X_2^{(n)}$: Bandwidth occupancy (Mbps)
- $X_3^{(n)}$: Transmission delay (ms)

The simulation considers the periods of the day when the network load is highest, specifically peak hours (8-10h) and (18-20h):

- In the deterministic model, the allocation of radio resources (X_1), increases progressively up to 90% at peak times, then decreases outside these periods; in the stochastic model, unpredictable oscillations due to fluctuations in user demand and radio interference, Thus we have: Random load peaks which vary the allocation between 85 and 95%, at peak times, more instability is observed in the evening (20-24h), which can affect network management.
- At the level of the metric (X_2), the deterministic model shows that bandwidth is at maximum utilization of 75 Mbps at peak times, indicating stable and predictable use of the radio channel, which is not the case at the stochastic model level, marked by a fluctuation of ±10 Mbps, impacting latency and network congestion. Between 18:00 and 20:00, traffic becomes irregular, resulting in packet losses and degraded QoS. However, unexpected decreases in the middle of the day indicate possible underutilization of resources.
- The transmission delay (X_3), a very important QoS metric in LTE technology, shows a maximum latency of 100-120 ms at peak times in the deterministic model, the response time is stable outside peak loads (≈40-50 ms), on the other hand, in the stochastic model, the observation gives a fluctuating delay, with peaks of 130-140 ms at peak times, which impacts VoLTE and streaming performance; In the evening (20-24h), latency is more unstable, probably due to unpredictable traffic and in the off-peak (0h-6h), we have less latency, indicating better resource allocation.

Discussions

Table (1) shows that the stochastic model exhibits greater variability, especially at peak times, as radio resources fluctuate by ±5%, which impacts dynamic allocation and transmission delay (+10%) on average, due to random congestion not captured by the deterministic model, thus degrading QoS stability in the network.

Table (2) compares the performance of the deterministic and stochastic models over the course of a day. In the off-peak period (0-6h), the two models are very close, as there are few traffic variations. At peak times and in the evening, bandwidth and delay are more unstable in the stochastic model due to random congestion; and in the evening it reveals a more irregular use of resources, suggesting a need for dynamic optimization.

The application of 4th-order Runge-Kutta differential equations (RK4) in modeling QoS optimization in LTE networks has enabled stable and accurate numerical integration over 24 hours, offering in-depth analysis of network dynamics under deterministic and stochastic approaches. The deterministic model, RK4, showed a maximum resource allocation of 90% at peak times (8-10 am and 6-8 pm), with a stabilized bandwidth of 75 Mbps and a fixed latency of 120 ms, illustrating predictable but limited management in the face of real traffic variations. Conversely, the stochastic-based approach revealed ±5% oscillations in resource allocation, ±10 Mbps variations in bandwidth, and fluctuating latency between 130 and 140 ms, demonstrating the need for dynamic resource adaptation to manage unforeseen congestion. RK4 thus enabled us to assess the robustness of the deterministic model, which is suitable for static network planning, and the realism of the stochastic model, which better reflects random fluctuations in user traffic. However, these simulations underline the fact that, despite the accuracy of RK4, effective QoS optimization in LTE networks requires dynamic resource adjustment, paving the way for the integration of predictive AI models to anticipate and manage congestion in real-time.

Table 1: Overall performance comparison between deterministic and stochastic

Parameter	Deterministic model	Stochastic model	Difference (%)	
X_1	$\overline{X_1}$: average allocation	60 %	62%	+3.3%
	Peak time allocation	90 %	85%-95%	Variability of ±5%
X_2	$\overline{X_2}$: Average bandwidth	45 Mbps	48 Mbps	+6.7%
	Peak-time bandwidth	75 Mbps	70-80 Mbps	Variability of ±10 Mbps
X_3	$\overline{X_3}$: Average delay	65 ms	72 ms	+10.8%
	Peak hour delay	120 ms	130-140 ms	Variability of ±10-20 ms

Table 2: Performance comparison between Deterministic (D) and Stochastic (S) according to Period of the Day

Period		X_1 (%)		X_2 (Mbps)		X_1(ms)	
Period	Hour	D	S	D	S	D	S
Night	(0-6h)	30	32	20	22	30	35
Morning	(6-8h)	50	53	35	38	50	55
Peak hour	Morning (8-10h) Evening (18-20h)	90	85-95	75	70-80	120	130-140
Day	(10-18h)	50	52	40	43	45	50
Evening	(20-24h)	40	45	30	35	40	45

Conclusion

The results obtained in this study highlight the value of the Runge-Kutta 4th-order (RK4) approach for optimizing Quality of Service (QoS) in an eUTRAN LTE network. By modeling the dynamic evolution of radio resources, bandwidth occupancy, and transmission delay, RK4 provides a better understanding of network

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variations and facilitates the adjustment of resources according to traffic conditions. Comparative analysis of the deterministic and stochastic approaches showed that the former offers a stable, predictable view of the network. However, this approach does not consider random fluctuations in traffic, which limits its effectiveness in the face of unforeseen congestion. Conversely, the stochastic approach, by incorporating random variations, provides a more realistic model of the network, offering greater adaptability in real-time. These results demonstrate the need for dynamic resource management based on intelligent congestion prediction. In the future, the integration of Machine Learning and time series could make it possible to anticipate periods of heavy load and automatically adjust resource allocation to improve the resilience and efficiency of the LTE network. In this way, a hybrid approach combining mathematical modeling (RK4) and artificial intelligence could provide an advanced solution for optimal, predictive management of next-generation mobile networks.

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Author's Contributions

Brou Pacôme: Author of the central concepts of the research article. His expertise in Applied Mathematics and Telecom was essential for the selectivity of QoS state variables in LTE technology such as radio resource allocation, bandwidth occupancy, and transmission delay based on his experience in mobile network optimization. The corresponding author of the paper helped in the application of Runge Kutta to order 4 in an LTE network, defining the input parameters of the Mathematical model at t = 0 and the Interpretation of the Results: Stochastic vs. Deterministic.

Pandry Ghislain: Researcher and computer resource provider for simulations. His expertise in differential equations was decisive for Runge Kutta's 4th-order modeling of the differential equation system to improve QoS in LTE Networks.

Bodjré Aka Hugues Félix: Python programming of deterministic models and stochastic variability to elucidate theoretical content. He helped define initial conditions and input parameters for LTE Model Simulation.

Oumtanaga Souleymane: Coordinator of all editorial content and point of contact between all authors. He helped develop the "Analysis and interpretation of results and discussions" and "Conclusion" sections.

Ethics

The authors do not foresee any ethical issues that may arise because of the publication of this manuscript.

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