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Research Article

Shift of Zeeman Components of Ionic Spectral Lines Caused by the Angular Doppler Effect

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Abstract: The angular Doppler effect was introduced by Garetz. While the usual Doppler effect is caused by the translational motion of the emitter, the angular Doppler effect is caused by the rotational motion of the emitter. It results into the frequency shift of a circularly-polarized light. In the present paper we analyze the angular Doppler effect caused by the rotational motion of ions in a magnetic field. We demonstrate that for the spectral lines emitted by ions, this effect results in the shift of the σ-components of the Zeeman triplet observed along the magnetic field. We compare the angular Doppler shift in this situation with the fine structure splitting. We show (by the example of He^+) that the angular Doppler shift can exceed the fine structure splitting at sub-GigaGauss magnetic fields. Magnetic fields of this magnitude and even much higher were theoretically expected to develop and were indeed measured in experiments studying relativistic laser-plasma interactions. Magnetic fields of this magnitude and even much higher occur in neutron stars, some of them having helium in their tiny atmospheres. These are the physical situations relevant to our findings.

Keywords: Angular Doppler Effect, Ionic Spectral Lines, Magnetic Field, Zeeman Components, Additional Doppler Shift, Magnetized Plasmas

Introduction

The usual (translational) Doppler effect was studied in numerous papers throughout the years. For examples we suggest (Alchemy, 2023; Andreev, 2011; Belyaev *et al.*, 2008; Censor, 1973; Censor, 1984; Censor and Vine, 1984; Cochran, 1989; Dingle, 1960; Giuliani, 2013; Gordienko *et al.*, 2004; Huang and Lu, 2004; Klinaku, 2016; Klinaku and Berisha, 2019; Mandelberg and Witten, 1962; Nolte, 2020; Neipp *et al.*, 2003) (listed in the alphabetical order) and references therein.

There exists also the angular Doppler effect. It was introduced by Garetz (1981). While the usual Doppler effect is caused by the translational motion of the emitter, the angular Doppler effect is caused by the rotational motion of the emitter. It results into the frequency shift of a circularly-polarized light by:

$$\Delta\omega_{AD} \approx \pm\Omega \tag{1}$$

where, Ω is the frequency of the rotation of the emitter – see Appendix A of the present paper.

In the present paper we consider the angular Doppler effect caused by the rotational motion of ions in a magnetic field. It can manifest as the shift of Zeeman components of spectral lines emitted by the ions. We show that this shift can significantly exceeds the fine structure splitting in the conditions of neutron stars.

Results

We consider a radiating ion (emitter) of mass M in a magnetic field B: Specifically – for the simplicity of formulas – a hydrogenic ion of the nuclear charge Z. The corresponding angular Doppler shift is equal to its frequency of the rotation in the magnetic field (according to Eq. (1)):

$$\Delta\omega_{AD} \approx \pm e\,(Z-1)\,B/\,(Mc) \tag{2}$$

where, e is the absolute value of the electron charge and c is the speed of light.

The σ-components of the Zeeman triplet observed along the magnetic field are circularly-polarized. Therefore, they would exhibit the shift given by Eq. (2). The choice of the sign in Eq. (2) depends on whether the light is observed parallel or anti-parallel to the magnetic field.

Let us compare this shift with the fine structure splitting:

$$\Delta\omega_{FS} \sim \alpha^2 Z^4 m_e e^4/\left(2\hbar^3 n^5\right) \tag{3}$$

where, α is the fine structure constant, m_e is the electron mass and n is the principal quantum number. By comparing Eqs. (2-3), it is easy to find out that the angular Doppler shift exceeds the fine structure splitting when the magnetic field exceeds the following critical value:

$$B_{cr} = \alpha^2 Z^4 m_e e^3 Mc / \left[2\hbar^3 n^5 e (Z-1)\right] \tag{4}$$

Figure (1) shows the 3-D plot of the dependence of B_{cr}(in MegaGauss) on the nuclear charge Z and on the principal quantum number n.

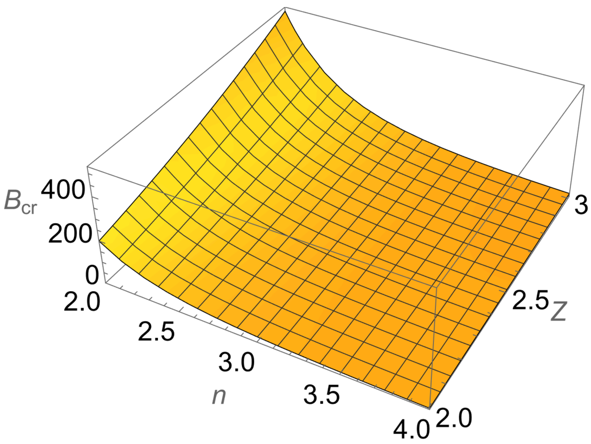

Fig. 1: The 3-D plot of the dependence of the critical magnetic field B_{cr} *(in MegaGauss) on the nuclear charge Z and on the principal quantum number n*

For example, for He^+ ($Z = 2$, $M = 4M_p$, where $M_p$is the proton mass), one obtains:

$$B_{cr} \sim \left(7/n^5\right) GigaGauss \tag{5}$$

From Figure (1) and from Eqs. (4-5), it is seen that B_{cr} very rapidly decreases as n increases. For $n = 2$, which is the smallest value of n relevant to the fine structure splitting, Eq. (5) yields:

$$B_{cr} \sim 200\, MegaGauss \tag{6}$$

Discussion

We analyzed the angular Doppler effect caused by the rotational motion of ions in a magnetic field. We showed that for the spectral lines emitted by ions, this effect results in the shift of the σ-components of the Zeeman triplet observed along the magnetic field.

We compared the angular Doppler shift in this situation with the fine structure splitting. We showed (by the example of He^+) that the angular Doppler shift can exceed the fine structure splitting at sub-GigaGauss magnetic fields.

Conclusion

Magnetic fields of this magnitude and even much higher were theoretically expected to develop and were indeed measured in experiments studying relativistic laser-plasma interactions – see, e.g., papers (Potekhin, 2014; Prokhovnik, 1980; Seddon and Bearpark, 2003; Shenar *et al.*, 2023; Sher, 1968) and references therein. Magnetic fields of this magnitude and even much higher occur in neutron stars, some of them having helium in their tiny atmospheres – see, e.g., (Tatarakis *et al.*, 2002a-b; Wagner *et al.*, 2004; Wattles, 2023) and references therein. These are the physical situations relevant to our findings.

Appendix A. Garetz (1981) derivation of Eq. (1)

Garetz (1981) considered a particle rotating with angular frequency Ω. If a circularly-polarized photon is emitted perpendicular to the rotation plane, then from the conservation of the total angular momentum L follows that $\Delta L = \pm\hbar$. The rotational kinetic energy is $E = L^2/(2I)$, I being the moment of inertia of the particle. From the energy conservation follows:

$$L_I^2/(2I) + E_1 = L_2^2/(2I) + E_2 + \hbar(\omega + \Delta\omega) \tag{A.1}$$

In Eq. (1), $E_{1,2}$ is the internal electronic energy of the radiating particle (the subscripts 1 and 2 mean before and after the emission, respectively), ω is the radiation frequency of the particle at rest and Δω is the rotation-caused angular Doppler shift.

From Eq. (A.1) it is easy to find that:

$$\hbar\Delta\omega = (\Delta L/I)(L_1 + L_2)/2 \tag{A.2}$$

where, $\Delta L = L_1 - L_2$ is supposed to be much smaller than min (L_1, L_2) by the absolute value, so that $(L_1 + L_2)/2 \approx L$ and:

$$\hbar\Delta\omega \approx= (L/I)\Delta L \tag{A.3}$$

Since $\Delta L = \pm\hbar$ and $L = I\Omega$, then from Eq. (A.3) follows:

$$\Delta\omega \approx= \pm\Omega \tag{A.4}$$

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Data Availability Statement

All data is included in the paper.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that no ethical issues involved.

Conflict of Interest

The author declares no conflict of interest.

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